CS222: Computer Architecture

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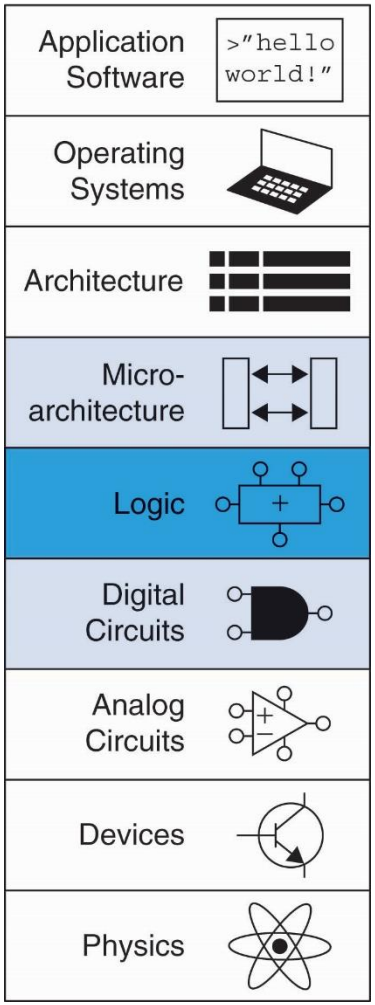
Chapter 5 :: Topics

- **Introduction**
- **Arithmetic Circuits**
- **Number Systems**
- **Sequential Building Blocks**
- **Memory Arrays**
- **Logic Arrays**

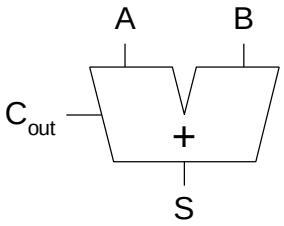


Introduction

- **Digital building blocks:**
  - Gates, multiplexers, decoders, registers, arithmetic circuits, counters, memory arrays, logic arrays
- **Building blocks demonstrate hierarchy, modularity, and regularity:**
  - Hierarchy of simpler components
  - Well-defined interfaces and functions
  - Regular structure easily extends to different sizes
- **Will use these building blocks in Chapter 7 to build microprocessor**

1-Bit Adders

Half Adder

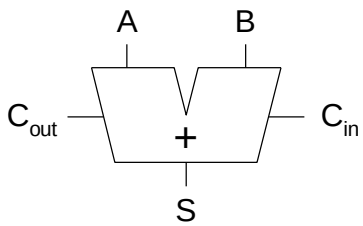


A	B	C_{out}	S
0	0		
0	1		
1	0		
1	1		

S =

C_{out} =

Full Adder



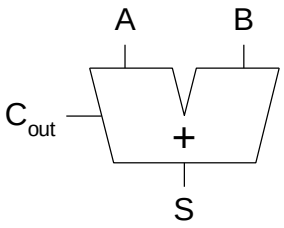
C_{in}	A	B	C_{out}	S
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

S =

C_{out} =

1-Bit Adders

Half Adder

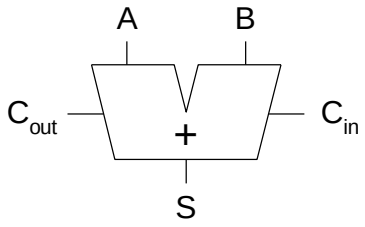


A	B	C_{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

S =

C_{out} =

Full Adder



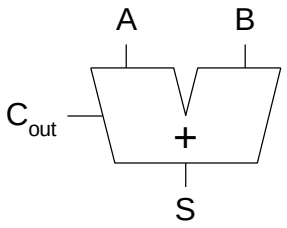
C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

S =

C_{out} =

1-Bit Adders

Half Adder

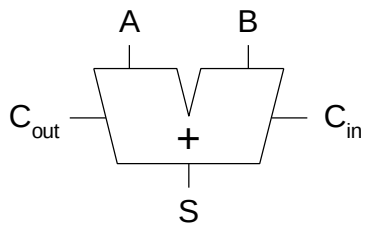


A	B	C_{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$S = A \oplus B$

$C_{out} = AB$

Full Adder



C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

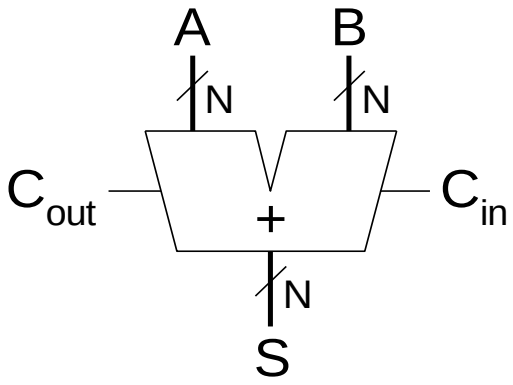
$S = A \oplus B \oplus C_{in}$

$C_{out} = AB + AC_{in} + BC_{in}$

Multibit Adders (CPAs)

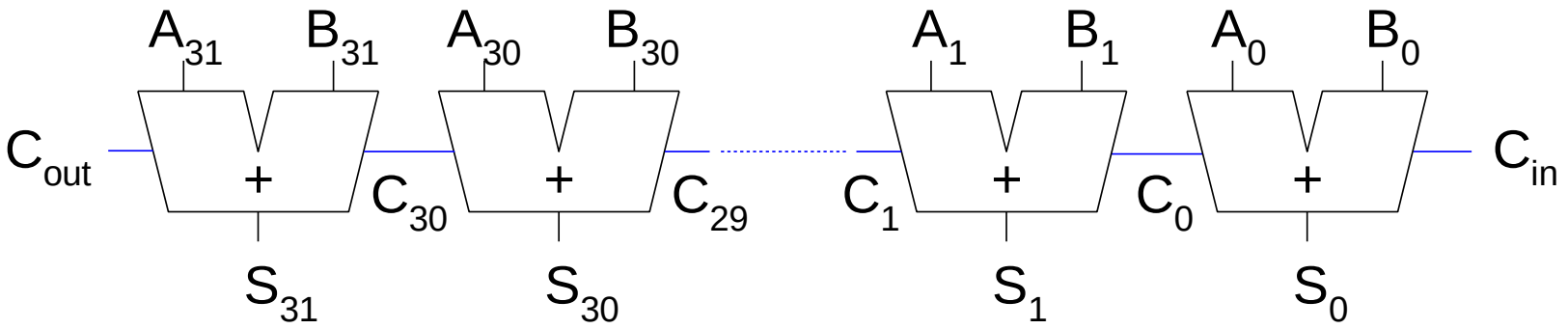
- Types of carry propagate adders (CPAs):
  - Ripple-carry (slow)
  - ~~Carry-lookahead (fast)~~
  - ~~Prefix (faster)~~
- Carry-lookahead and prefix adders faster for large adders but require more hardware

Symbol



Ripple-Carry Adder

- Chain 1-bit adders together
- Carry ripples through entire chain
- Disadvantage: **slow**



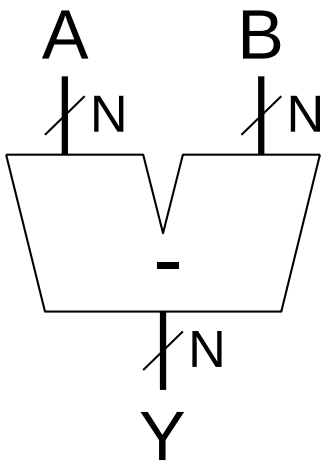
Ripple-Carry Adder Delay

$$t_{ripple} = Nt_{FA}$$

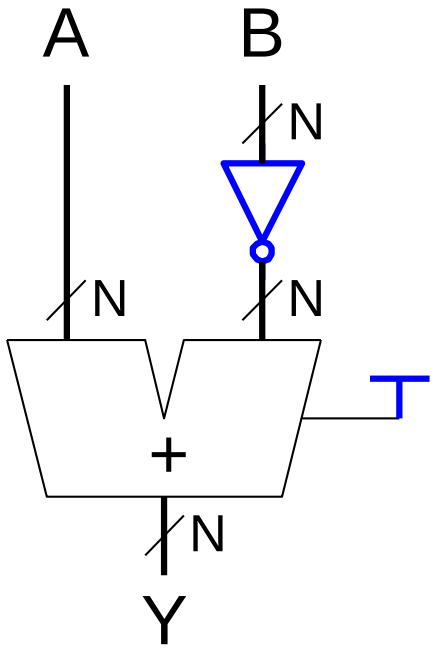
where t_{FA} is the delay of a 1-bit full adder

Subtractor

Symbol

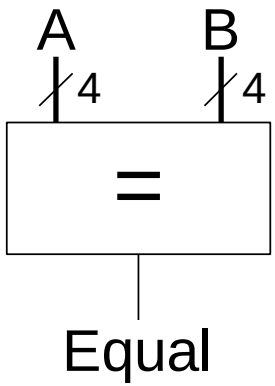


Implementation

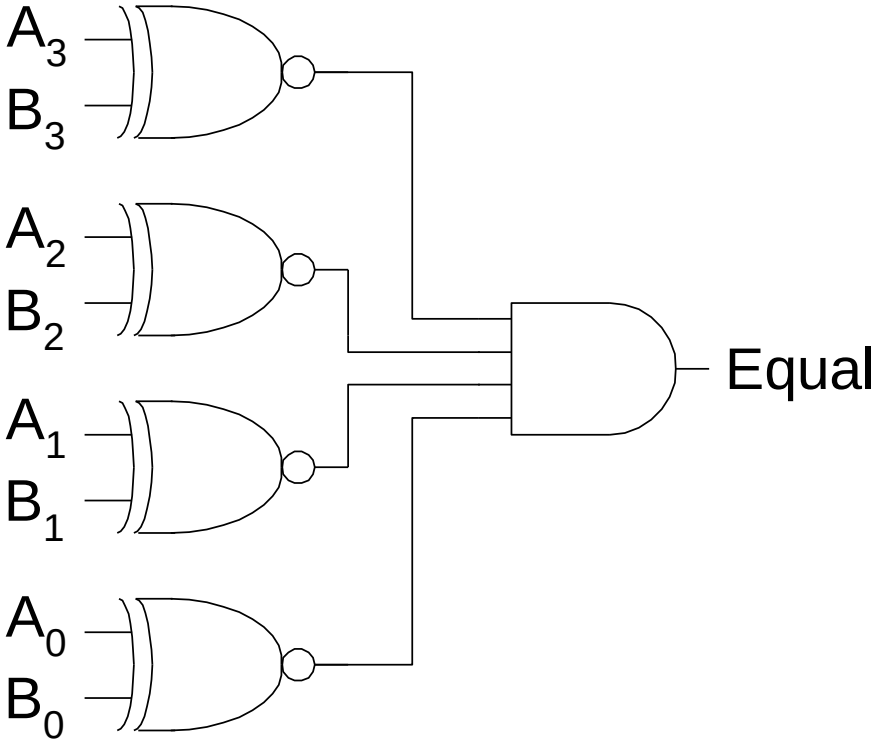


Comparator: Equality

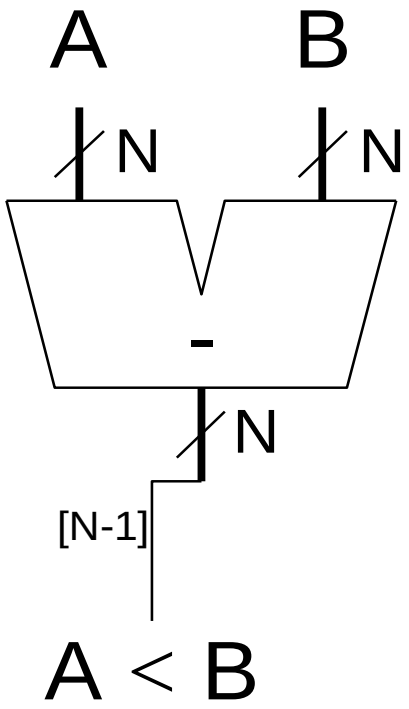
Symbol



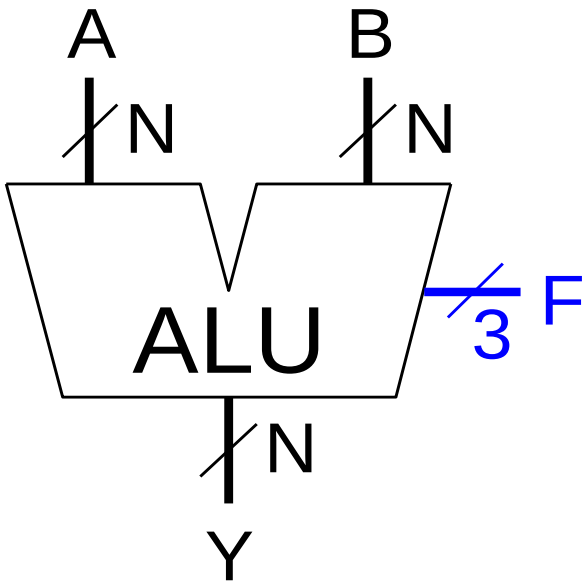
Implementation



Comparator: Less Than

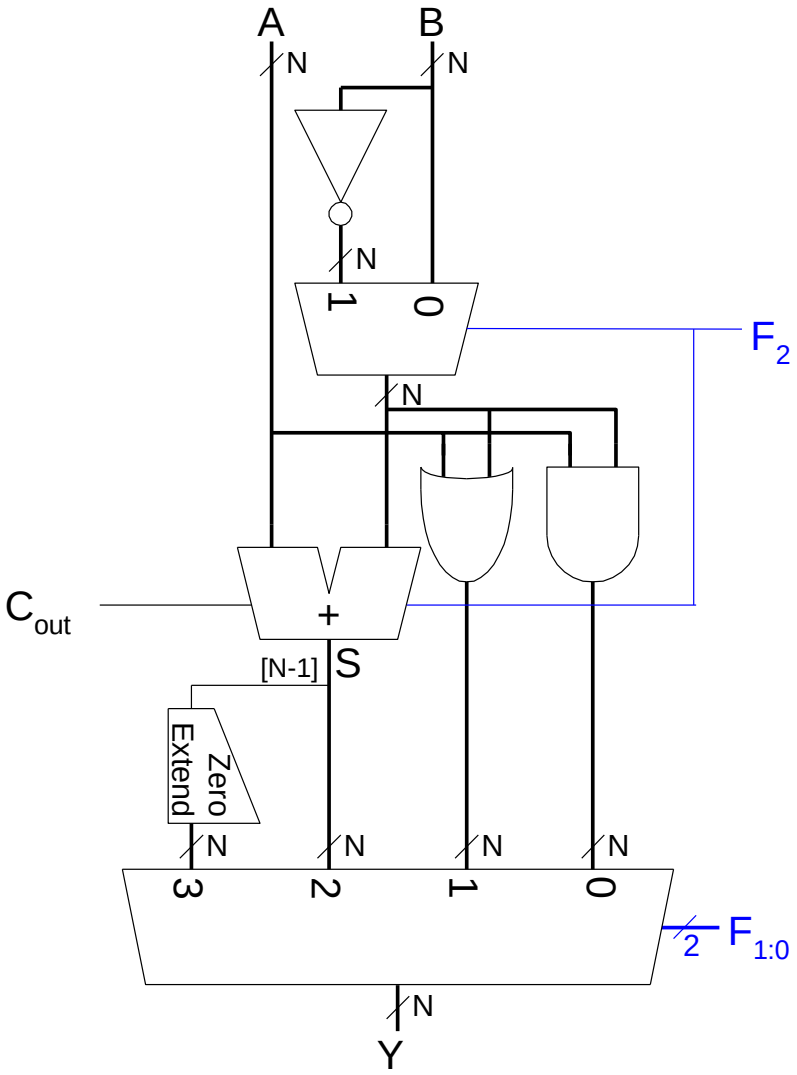


Arithmetic Logic Unit (ALU)



$F_{2:0}$	Function
000	A & B
001	A | B
010	A + B
011	not used
100	A & ~B
101	A | ~B
110	A - B
111	SLT

ALU Design



$F_{2:0}$	Function
000	A & B
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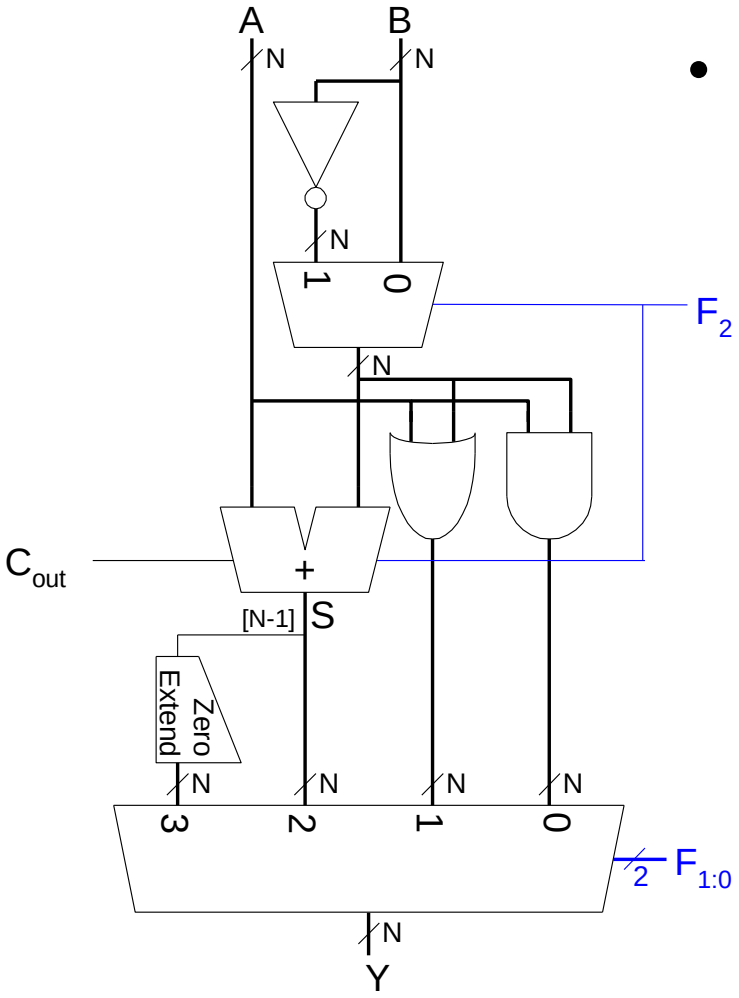
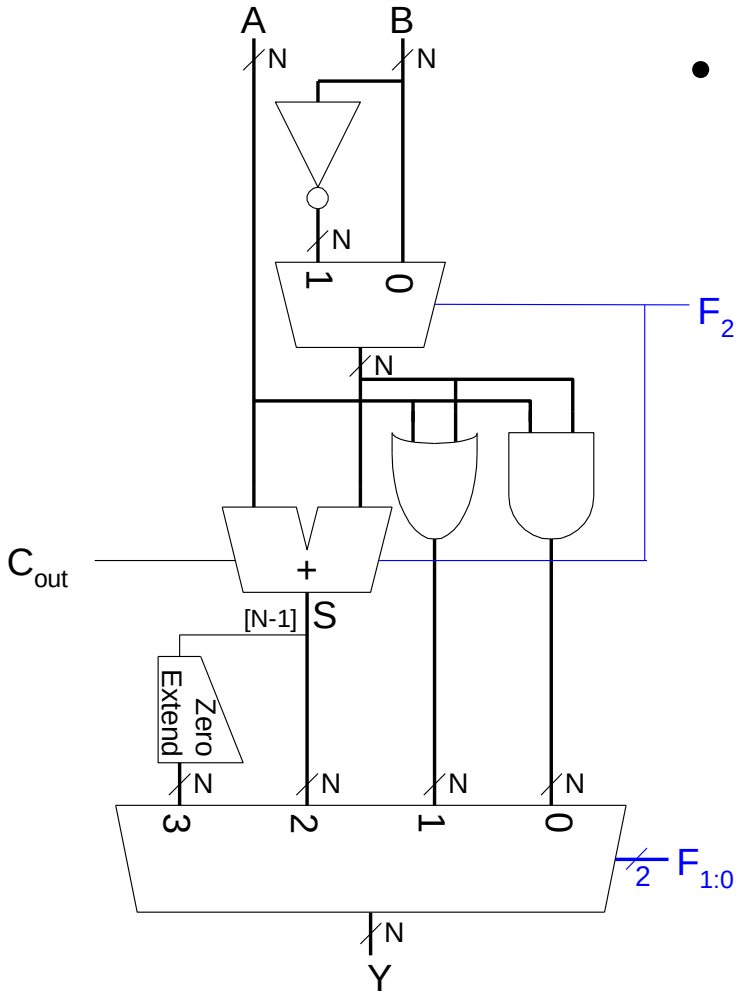
111	SLT

Set Less Than (SLT) Example



- Configure 32-bit ALU for SLT operation: $A = 25$ and $B = 32$

Set Less Than (SLT) Example



- Configure 32-bit ALU for SLT operation: $A = 25$ and $B = 32$
  - $A < B$, so Y should be 32-bit representation of 1 (0x00000001)
  - $F_{2:0} = 111$
    - $F_2 = 1$ (adder acts as subtracter), so 25 - 32 = -7
    - -7 has 1 in the most significant bit ($S_{31} = 1$)
    - $F_{1:0} = 11$ multiplexer selects $Y = S_{31}$ (zero extended) = 0x00000001.

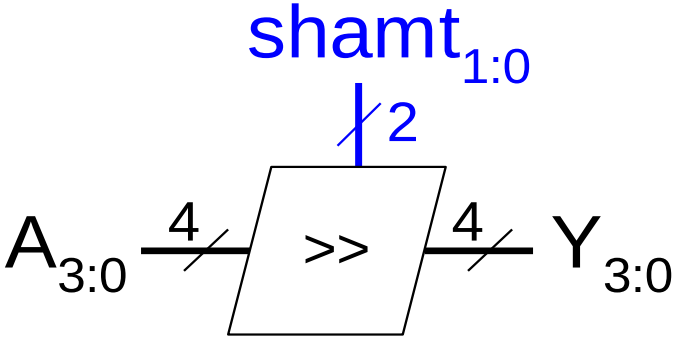
Shifters

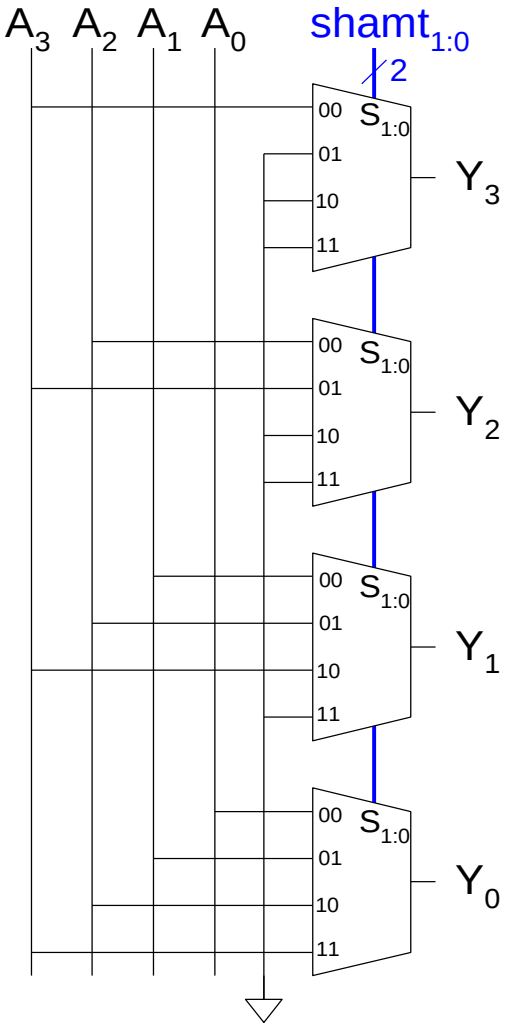
- **Logical shifter:** shifts value to left or right and fills empty spaces with 0's
  - Ex: 11001 >> 2 =
  - Ex: 11001 << 2 =
- **Arithmetic shifter:** same as logical shifter, but on right shift, fills empty spaces with the old most significant bit (msb).
  - Ex: 11001 >>> 2 =
  - Ex: 11001 <<< 2 =
- **Rotator:** rotates bits in a circle, such that bits shifted off one end are shifted into the other end
  - Ex: 11001 ROR 2 =
  - Ex: 11001 ROL 2 =

Shifters

- **Logical shifter:**
  - Ex: 11001 >> 2 = 00110
  - Ex: 11001 << 2 = 00100
- **Arithmetic shifter:**
  - Ex: 11001 >>> 2 = 11110
  - Ex: 11001 <<< 2 = 00100
- **Rotator:**
  - Ex: 11001 ROR 2 = 01110
  - Ex: 11001 ROL 2 = 00111

Shifter Design





Shifters as Multipliers, Dividers

- $A << N = A \times 2^N$
  - **Example:** 00001 << 2 = 00100 ($1 \times 2^2 = 4$)
  - **Example:** 11101 << 2 = 10100 ($-3 \times 2^2 = -12$)
- $A >>> N = A \div 2^N$
  - **Example:** 01000 >>> 2 = 00010 ($8 \div 2^2 = 2$)
  - **Example:** 10000 >>> 2 = 11100 ($-16 \div 2^2 = -4$)

Multipliers

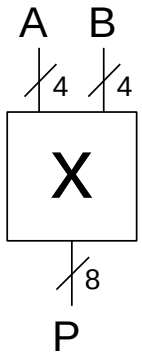
- **Partial products** formed by multiplying a single digit of the multiplier with multiplicand
- **Shifted** partial products **summed** to form result

Decimal		Binary
230	multiplicand	0101
x 42	multiplier	x 0111
460	partial products	0101
+ 920		0101
9660		0101
		+ 0000
	result	0100011

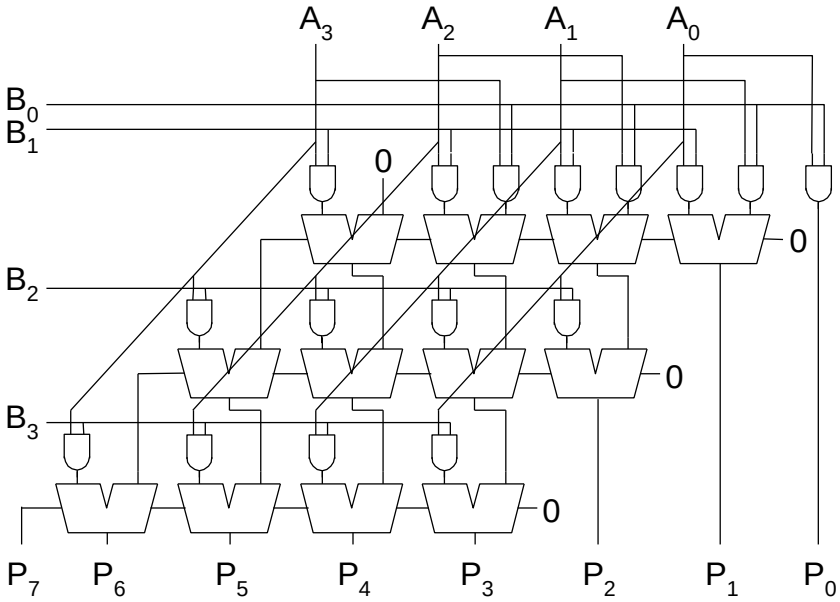
230 x 42 = 9660

5 x 7 = 35

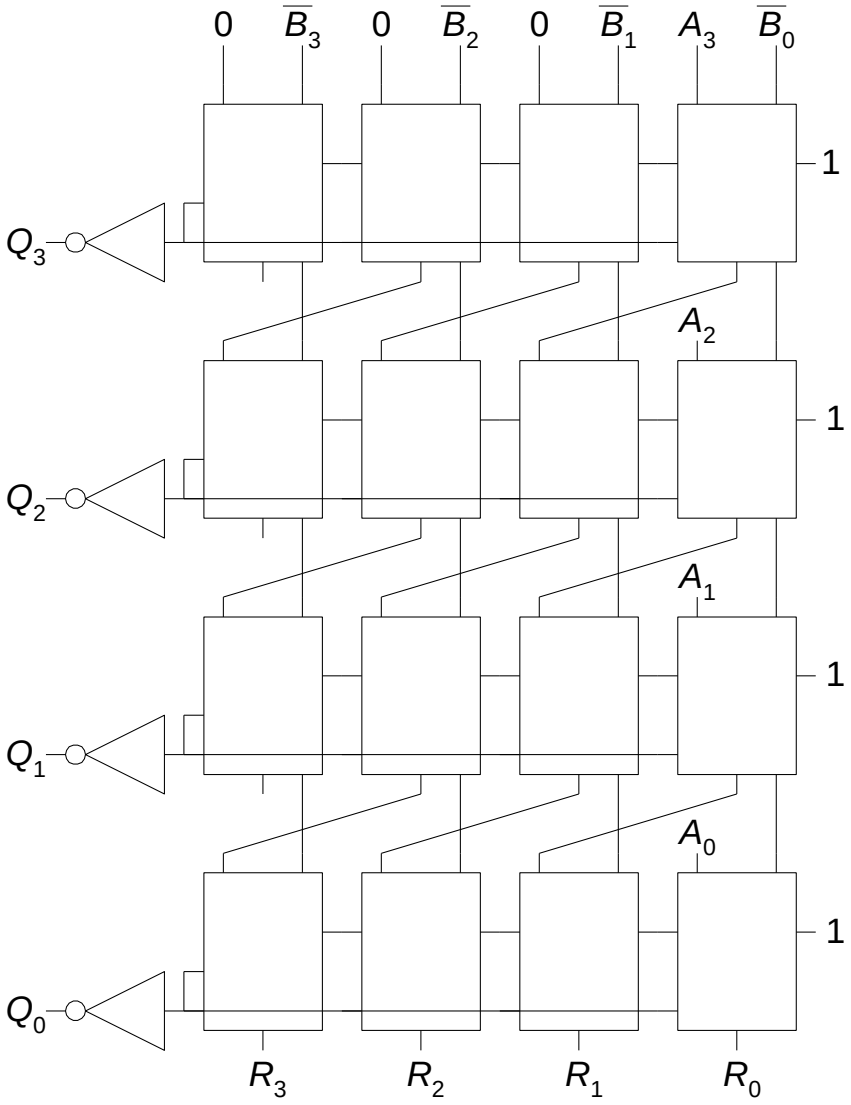
4 x 4 Multiplier



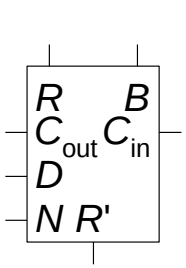
$$
\begin{array}{rcccccccc}
 & & & A_3 & A_2 & A_1 & A_0 & & \\
\times & & & B_3 & B_2 & B_1 & B_0 & & \\
\hline
 & & & A_3B_0 & A_2B_0 & A_1B_0 & A_0B_0 & & \\
 & & A_3B_1 & A_2B_1 & A_1B_1 & A_0B_1 & & & \\
 & A_3B_2 & A_2B_2 & A_1B_2 & A_0B_2 & & & & \\
+ & A_3B_3 & A_2B_3 & A_1B_3 & A_0B_3 & & & & \\
\hline
P_7 & P_6 & P_5 & P_4 & P_3 & P_2 & P_1 & P_0
\end{array}
$$

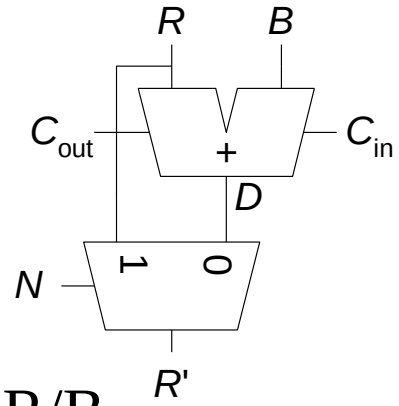


4 x 4 Divider



Legend





$$A/B = Q + R/B$$

```
R' = 0
for i = N-1 to 0
   R = {R' << 1, Ai}
   D = R - B
   if D < 0 then     Qi = 0, R' = R    // R < B
   else              Qi = 1, R' = D    // R ≥ B
R = R'
```

4 x 4 Divider

A/B = Q + R/B

Algorithm:

R′ = 0

for i = N-1 to 0

  R = {R′ << 1, A_i}

  D = R - B

  if D < 0, Q_i=0, R′ =R

  else Q_i=1, R′ =D

R′ =R

Example 9/3 =

A= 9 [1001], B = 3 [0011]

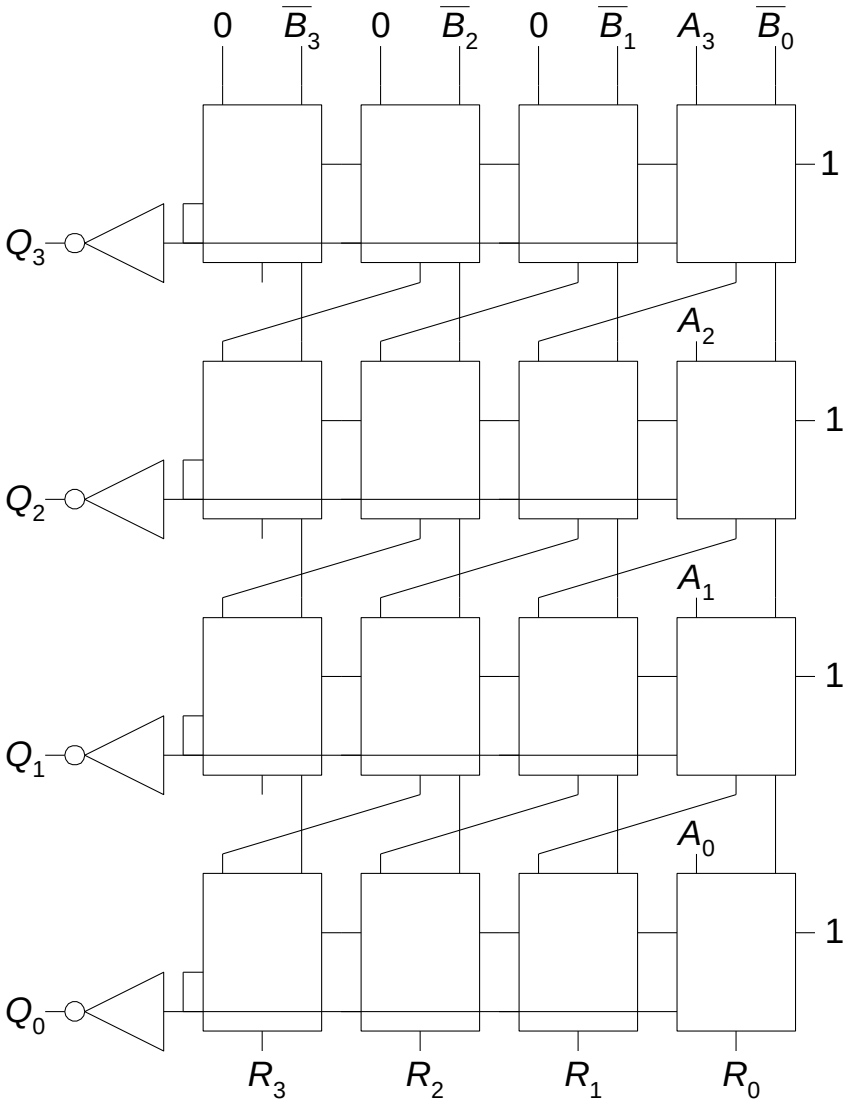
A_3= 1, A_2= 0, A_1= 0, A_0= 1

Iter (i)	R	D	Q	R′
3	{000,1}	1-3<0	0	0001
2	0010	2-3<0	0	0010
1	0100	4-3>0	1	0001
0	0011	3-3=0	1	0000

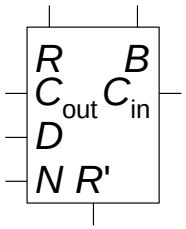
Q_3= 0, Q_2= 0, Q_1= 1, Q_0= 1

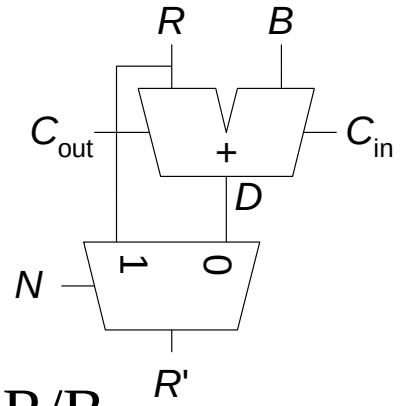
Q = 3 R′ = 0

4 x 4 Divider



Legend





A/B = Q + R/B

Algorithm:

R′ = 0

for i = N-1 to 0

  R = {R′ << 1. A_i}

  D = R - B

  if D < 0, Q_i=0, R′ =R

  else Q_i=1, R′ =D

R′ =R

Number Systems

- Numbers we can represent using binary representations
  - **Positive numbers**
    - Unsigned binary
  - **Negative numbers**
    - Two's complement
    - Sign/magnitude numbers

- What about **fractions**?

Numbers with Fractions

- Two common notations:
  - **Fixed-point:** binary point fixed
  - **Floating-point:** binary point floats to the right of the most significant 1

Fixed-Point Numbers

- 6.75 using 4 integer bits and 4 fraction bits:

$$01101100$$

$$0110.1100$$

$$2^2 + 2^1 + 2^{-1} + 2^{-2} = 6.75$$

  - Binary point is implied
  - The number of integer and fraction bits must be agreed upon beforehand

Fixed-Point Number Example

- Represent 7.5_{10} using 4 integer bits and 4 fraction bits.

Fixed-Point Number Example

- Represent 7.5_{10} using 4 integer bits and 4 fraction bits.

01111000

Signed Fixed-Point Numbers

- **Representations:**
  - Sign/magnitude
  - Two's complement
- **Example:** Represent -7.5_{10} using 4 integer and 4 fraction bits
  - **Sign/magnitude:**
  - **Two's complement:**

Signed Fixed-Point Numbers

- **Representations:**
  - Sign/magnitude
  - Two's complement
- **Example:** Represent -7.5_{10} using 4 integer and 4 fraction bits
  - **Sign/magnitude:**

    11111000
  - **Two's complement:**

1. +7.5:	01111000
2. Invert bits:	10000111
3. Add 1 to lsb:	+ 1
	10001000

Floating-Point Numbers

- Binary point floats to the right of the most significant 1
- Similar to decimal scientific notation

- For example, write 273_{10} in scientific notation:

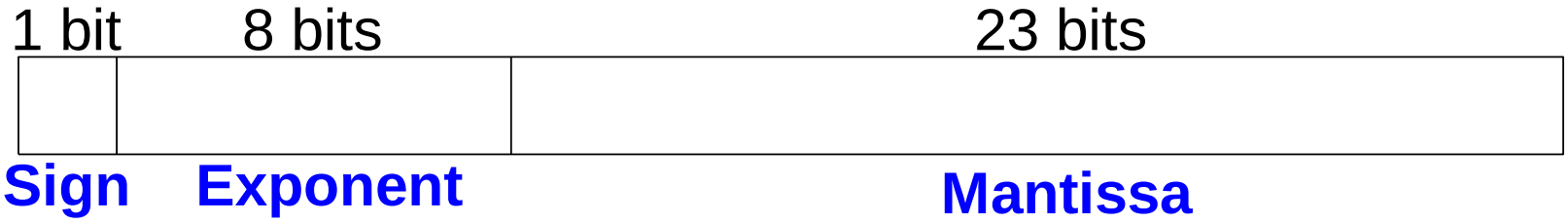
$$\mathbf{273 = 2.73 \times 10^2}$$

- In general, a number is written in scientific notation as:

$$\mathbf{\pm M \times B^E}$$

  - **M** = mantissa
  - **B** = base
  - **E** = exponent
  - In the example, M = 2.73, B = 10, and E = 2

Floating-Point Numbers



- **Example:** represent the value 228_{10} using a 32-bit floating point representation

We show three versions –final version is called the **IEEE 754 floating-point standard**

Floating-Point Representation 1

1. Convert decimal to binary (**don't reverse steps 1 & 2!**):

   $228_{10} = 11100100_2$

2. Write the number in "binary scientific notation":

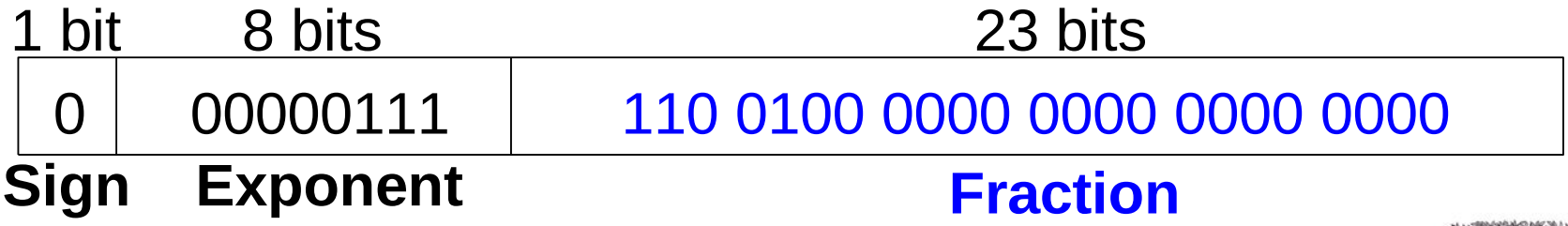
   $11100100_2 = 1.11001_2 \times 2^7$

3. Fill in each field of the 32-bit floating point number:
   - The sign bit is positive (0)
   - The 8 exponent bits represent the value 7
   - The remaining 23 bits are the mantissa

1 bit	8 bits	23 bits
0	00000111	11 1001 0000 0000 0000 0000
Sign	**Exponent**	**Mantissa**

Floating-Point Representation 2

- First bit of the mantissa is always 1:
  - $228_{10} = 11100100_2 = \mathbf{1}.11001 \times 2^7$
- So, no need to store it: *implicit leading 1*
- Store just fraction bits in 23-bit field

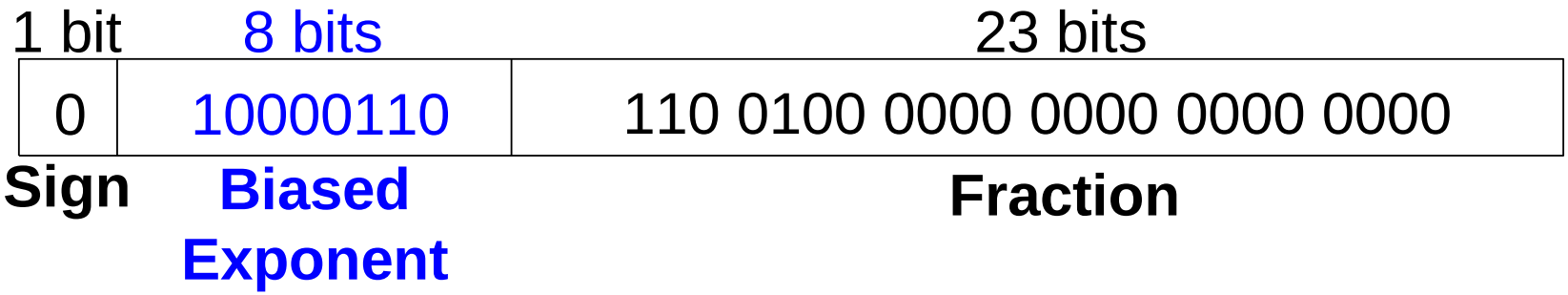


Floating-Point Representation 3

- *Biased exponent*: bias = 127 (01111111_2)
  - Biased exponent = bias + exponent
  - Exponent of 7 is stored as:

$$127 + 7 = 134 = 0x10000110_2$$

- The **IEEE 754 32-bit floating-point representation** of 228_{10}



in hexadecimal: **0x43640000**

Floating-Point Example

Write -58.25_{10} in floating point (IEEE 754)

1. Convert decimal to binary:

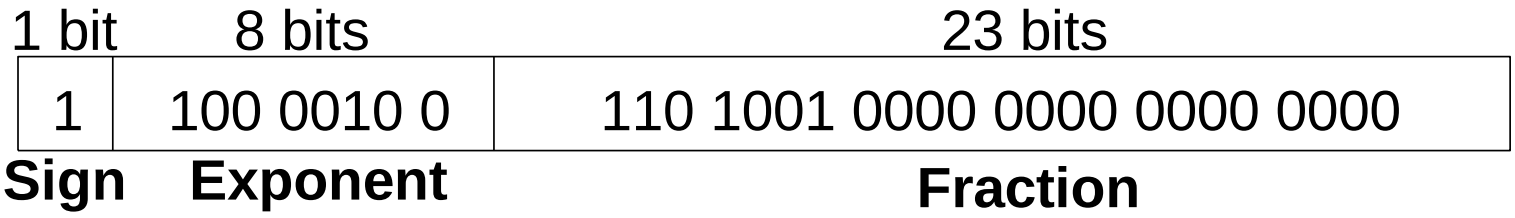
   $58.25_{10} = \mathbf{111010.01_2}$

2. Write in binary scientific notation:

   $\mathbf{1.1101001 \times 2^5}$

3. Fill in fields:

   Sign bit: 1 (negative)
   8 exponent bits: (127 + 5) = 132 = $\mathbf{10000100_2}$
   23 fraction bits: **110 1001 0000 0000 0000 0000**



in hexadecimal: **0xC2690000**

Floating-Point: Special Cases

Number	Sign	Exponent	Fraction
0	X	00000000	00000000000000000000000
∞	0	11111111	00000000000000000000000
- ∞	1	11111111	00000000000000000000000
NaN	X	11111111	non-zero

Floating-Point Precision

- **Single-Precision:**
  - 32-bit
  - 1 sign bit, 8 exponent bits, 23 fraction bits
  - bias = 127
- **Double-Precision:**
  - 64-bit
  - 1 sign bit, 11 exponent bits, 52 fraction bits
  - bias = 1023

Floating-Point: Rounding

- **Overflow:** number too large to be represented
- **Underflow:** number too small to be represented
- **Rounding modes:**
  - Down
  - Up
  - Toward zero
  - To nearest
- **Example:** round 1.100101 (1.578125) to only 3 fraction bits
  - Down: 1.100
  - Up: 1.101
  - Toward zero: 1.100
  - To nearest: 1.101 (1.625 is closer to 1.578125 than 1.5 is)

Floating-Point Addition

1. Extract exponent and fraction bits
2. Prepend leading 1 to form mantissa
3. Compare exponents
4. Shift smaller mantissa if necessary
5. Add mantissas
6. Normalize mantissa and adjust exponent if necessary
7. Round result
8. Assemble exponent and fraction back into floating-point format

Floating-Point Addition Example

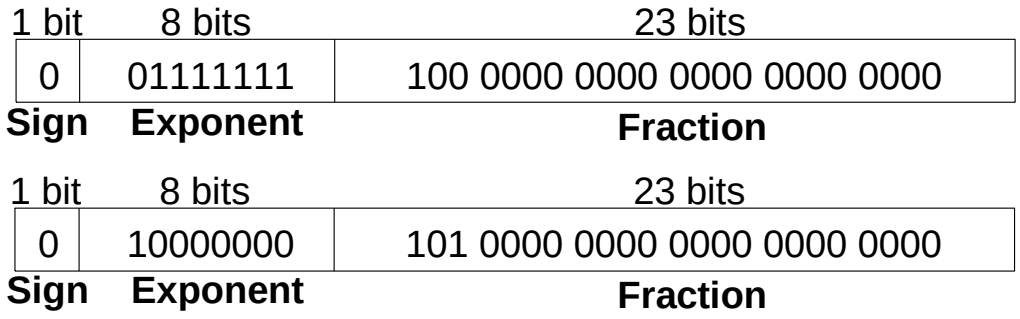
Add the following floating-point numbers:

0x3FC00000

0x40500000

Floating-Point Addition Example

1. Extract exponent and fraction bits



For first number (N1): S = 0, E = 127, F = .1

For second number (N2): S = 0, E = 128, F = .101

2. Prepend leading 1 to form mantissa

N1: 1.1

N2: 1.101

Floating-Point Addition Example

3. Compare exponents

127 – 128 = -1, so shift N1 right by 1 bit

4. Shift smaller mantissa if necessary

shift N1's mantissa: 1.1 >> 1 = 0.11 $(\times 2^1)$

5. Add mantissas

$$\begin{array}{r} 0.11 \times 2^1 \\ +\ 1.101 \times 2^1 \\ \hline 10.011 \times 2^1 \end{array}$$

Floating Point Addition Example

6. **Normalize mantissa and adjust exponent if necessary**

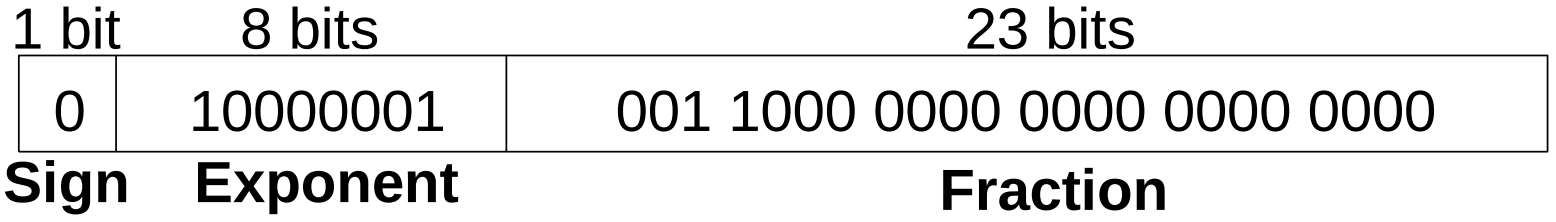
   $10.011 \times 2^1 = 1.0011 \times 2^2$

7. **Round result**

   No need (fits in 23 bits)

8. **Assemble exponent and fraction back into floating-point format**

   $S = 0, E = 2 + 127 = 129 = 10000001_2, F = 001100..$

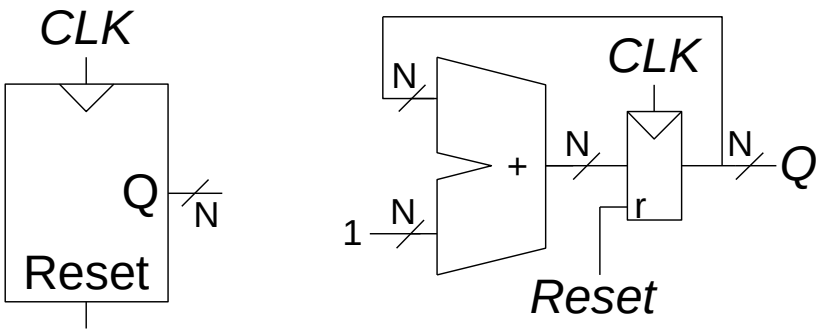


in hexadecimal: **0x40980000**

Counters

- Increments on each clock edge
- Used to cycle through numbers. For example,
  - 000, 001, 010, 011, 100, 101, 110, 111, 000, 001…
- Example uses:
  - Digital clock displays
  - Program counter: keeps track of current instruction executing

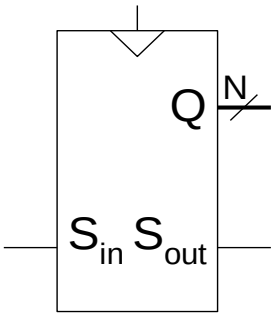
Symbol **Implementation**



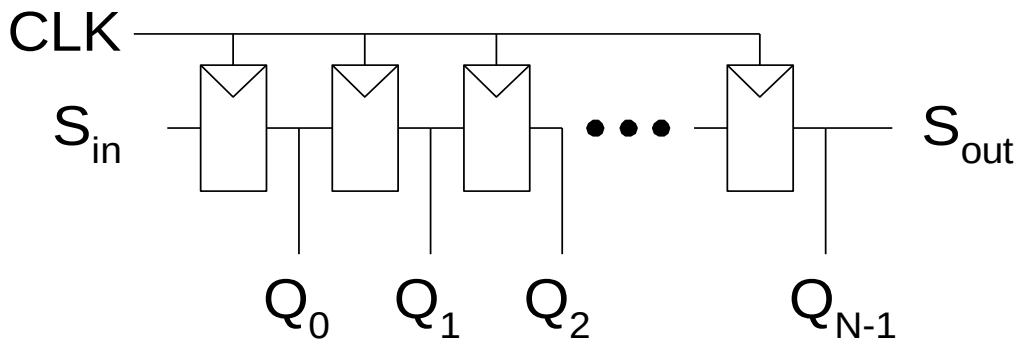
Shift Registers

- Shift a new bit in on each clock edge
- Shift a bit out on each clock edge
- *Serial-to-parallel converter*: converts serial input (S_{in}) to parallel output ($Q_{0:N-1}$)

Symbol:

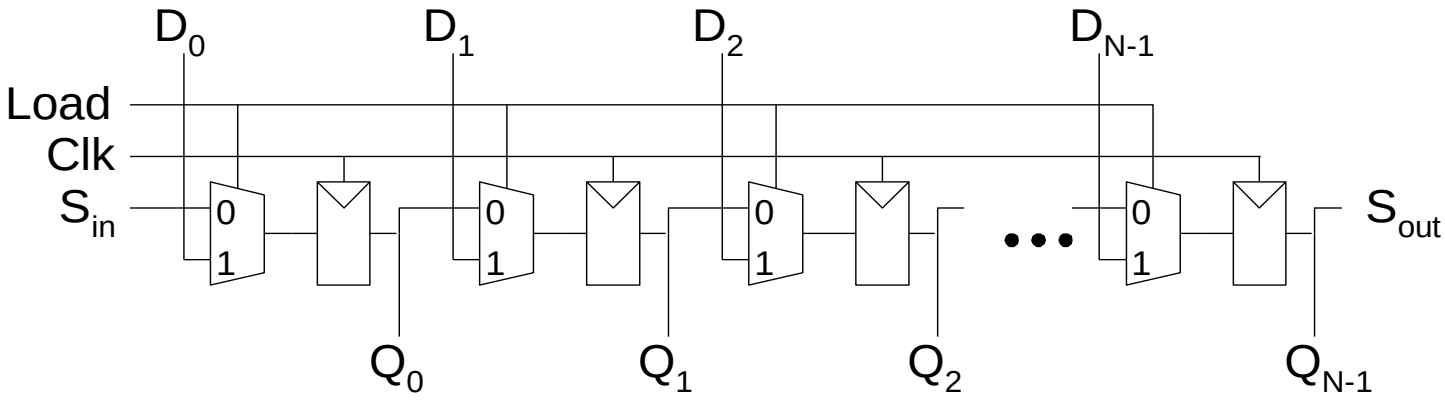


Implementation:



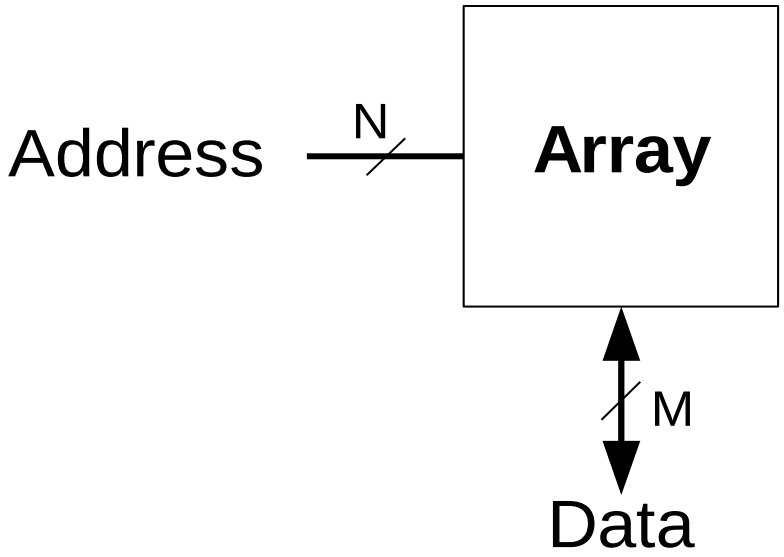
Shift Register with Parallel Load

- When *Load* = 1, acts as a normal *N*-bit register
- When *Load* = 0, acts as a shift register
- Now can act as a *serial-to-parallel converter* (S_{in} to $Q_{0:N-1}$) or a *parallel-to-serial converter* ($D_{0:N-1}$ to S_{out})



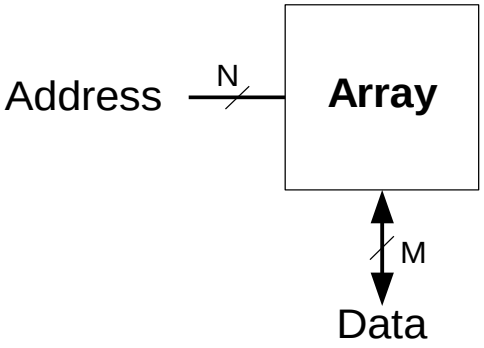
Memory Arrays

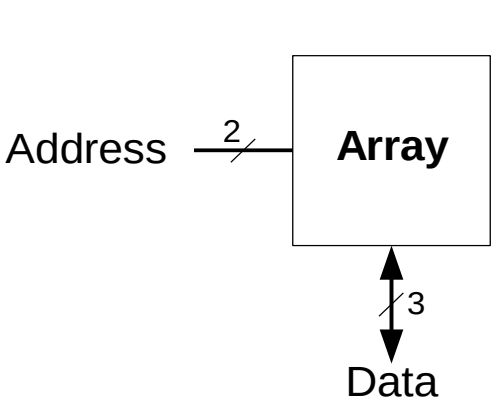
- Efficiently store large amounts of data
- 3 common types:
  - Dynamic random access memory (DRAM)
  - Static random access memory (SRAM)
  - Read only memory (ROM)
- *M*-bit data value read/ written at each unique *N*-bit address

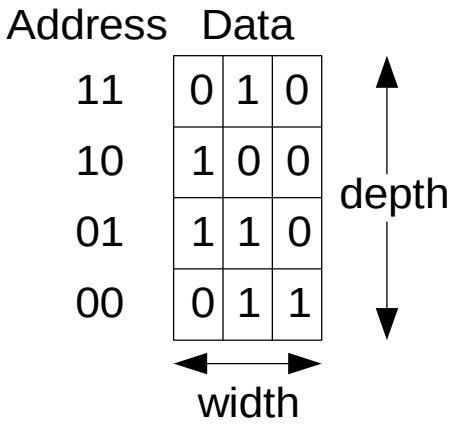


Memory Arrays

- 2-dimensional array of bit cells
- Each bit cell stores one bit
- N address bits and M data bits:
  - 2^N rows and M columns
  - **Depth:** number of rows (number of words)
  - **Width:** number of columns (size of word)
  - **Array size:** depth × width = $2^N \times M$

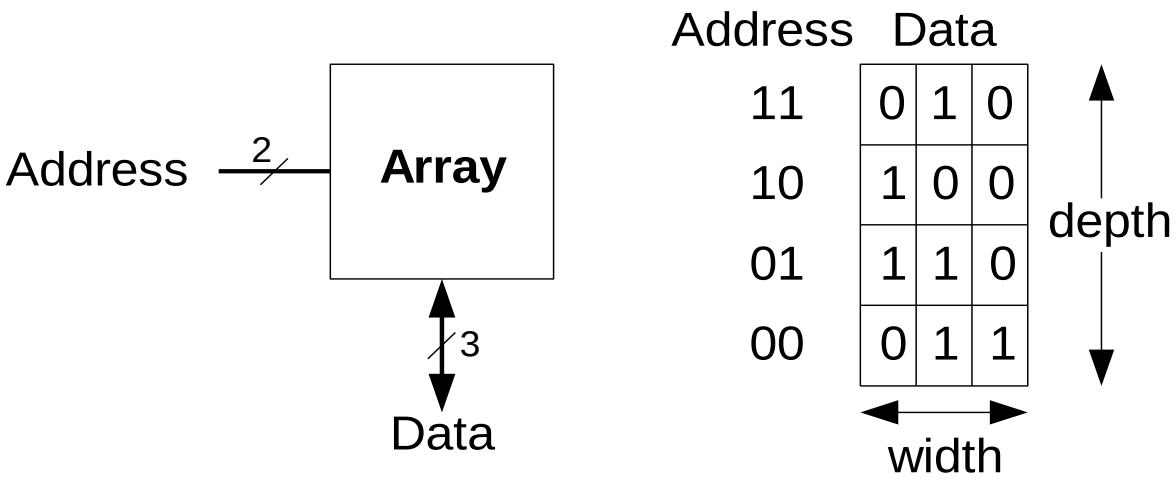




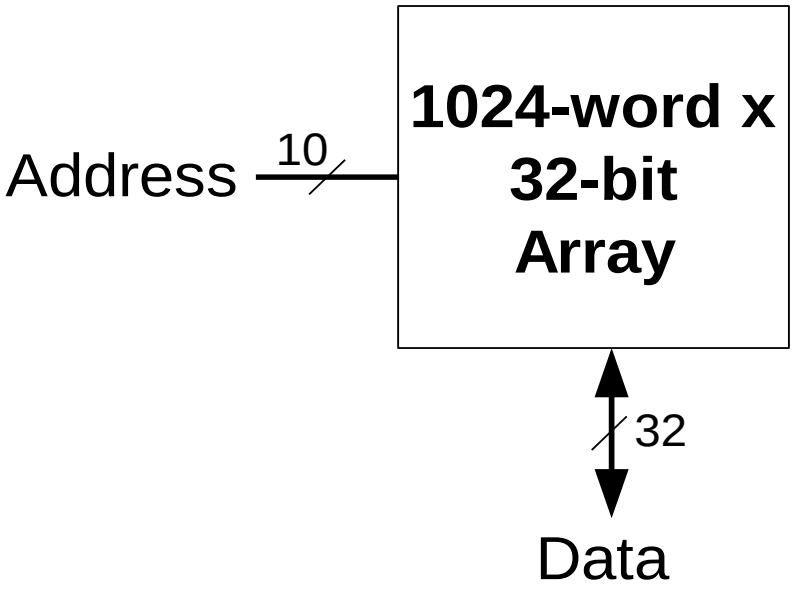


Memory Array Example

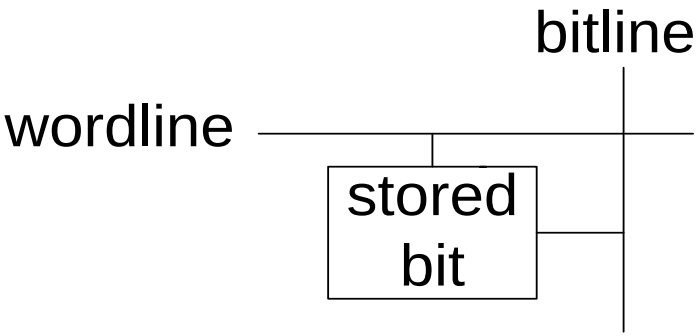
- $2^2 \times 3$-bit array
- Number of words: 4
- Word size: 3-bits
- For example, the 3-bit word stored at address 10 is 100

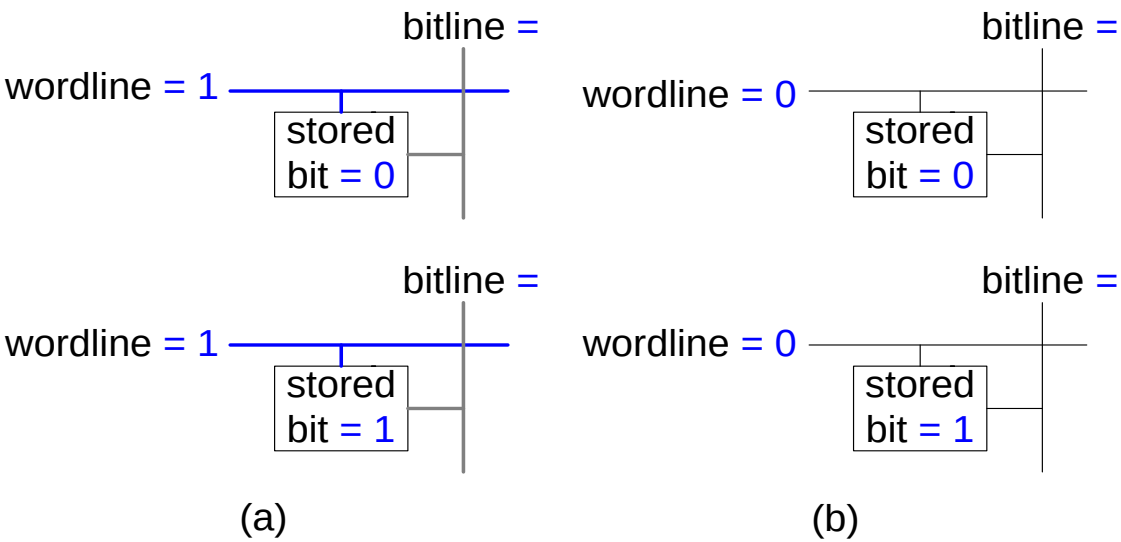


Memory Arrays

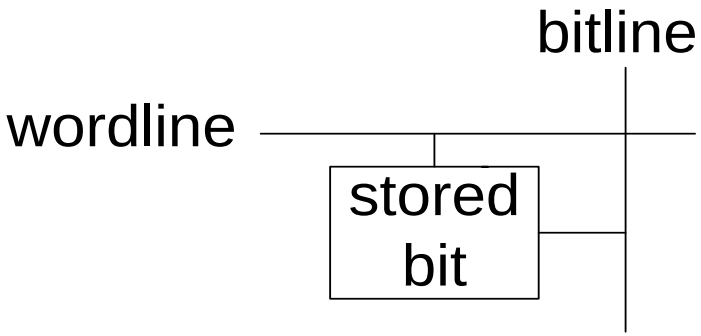


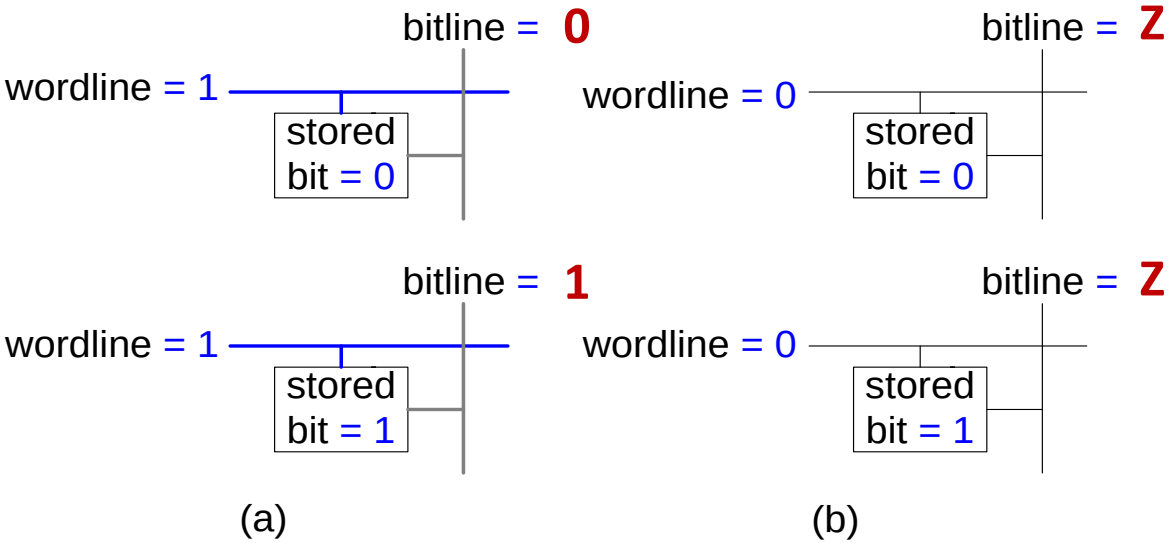
Memory Array Bit Cells





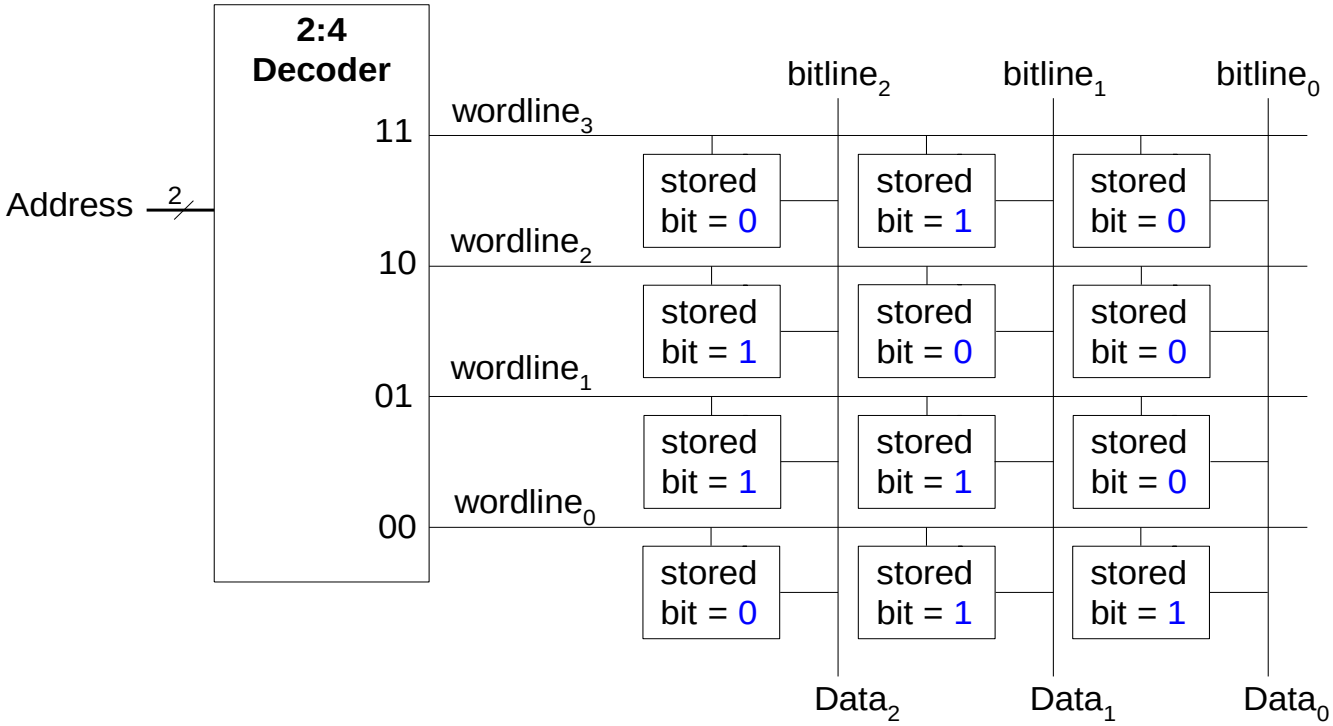
Memory Array Bit Cells





Memory Array

- **Wordline:**
  - like an enable
  - single row in memory array read/written
  - corresponds to unique address
  - only one wordline HIGH at once



Types of Memory

- Random access memory (RAM): **volatile**
- Read only memory (ROM): **nonvolatile**

RAM: Random Access Memory

- **Volatile:** loses its data when power off
- Read and written quickly
- Main memory in your computer is RAM (DRAM)

Historically called *random* access memory because any data word accessed as easily as any other (in contrast to sequential access memories such as a tape recorder)

ROM: Read Only Memory

- **Nonvolatile:** retains data when power off
- Read quickly, but writing is impossible or slow
- Flash memory in cameras, thumb drives, and digital cameras are all ROMs

Historically called *read only* memory because ROMs were written at manufacturing time or by burning fuses. Once ROM was configured, it could not be written again. This is no longer the case for Flash memory and other types of ROMs.

Types of RAM

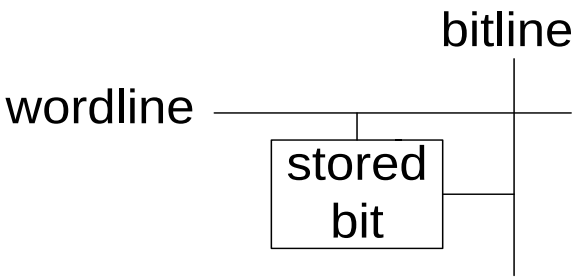
- **DRAM** (Dynamic random access memory)
- **SRAM** (Static random access memory)
- Differ in how they store data:
  - DRAM uses a capacitor
  - SRAM uses cross-coupled inverters

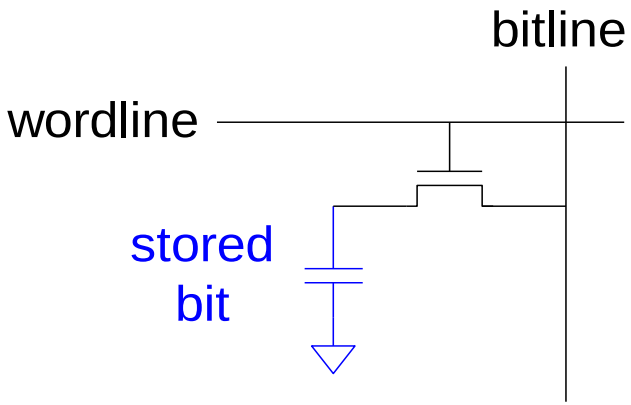
Robert Dennard, 1932 -

- Invented DRAM in 1966 at IBM
- Others were skeptical that the idea would work
- By the mid-1970's DRAM in virtually all computers

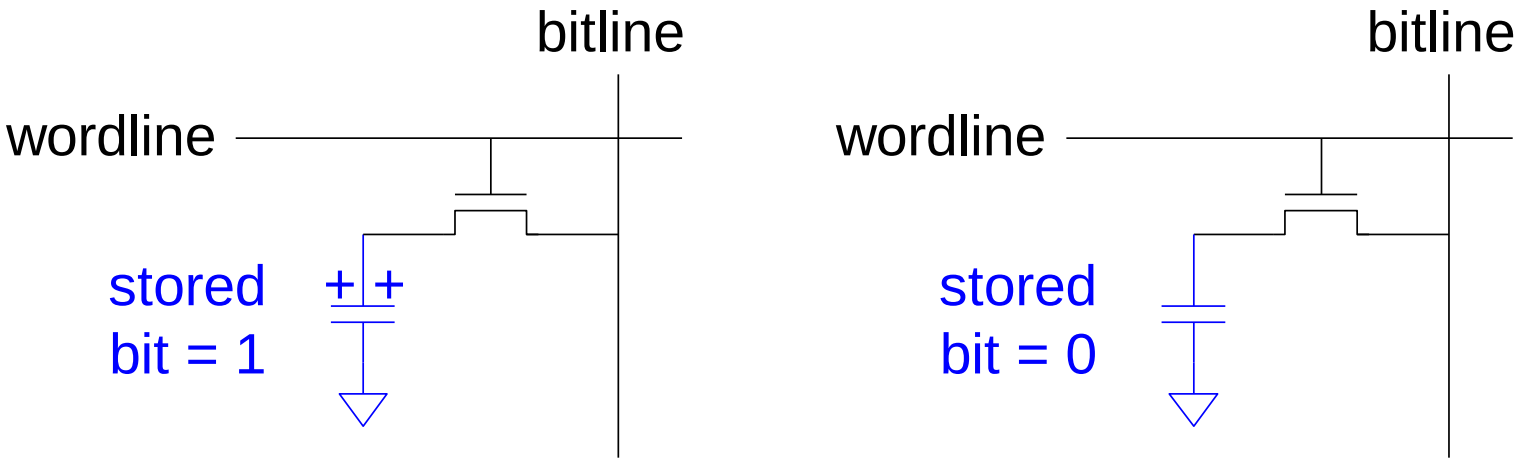
DRAM

- Data bits stored on capacitor
- *Dynamic* because the value needs to be refreshed (rewritten) periodically and after read:
  - Charge leakage from the capacitor degrades the value
  - Reading destroys the stored value

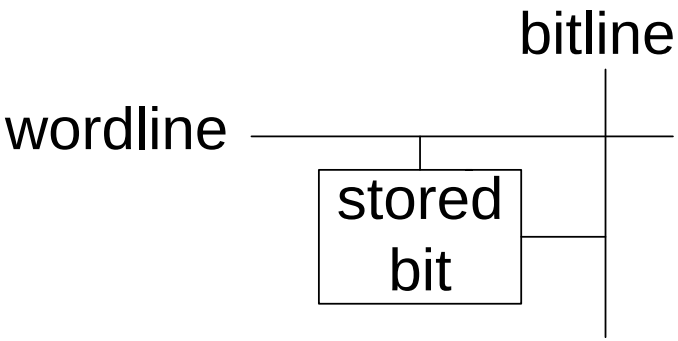


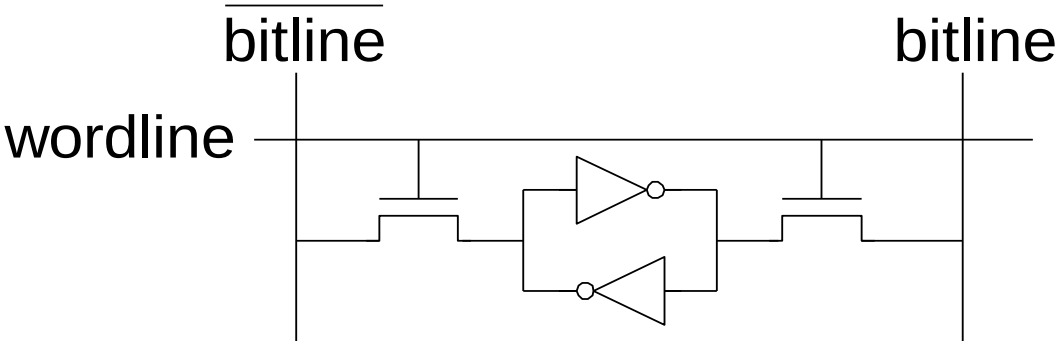


DRAM

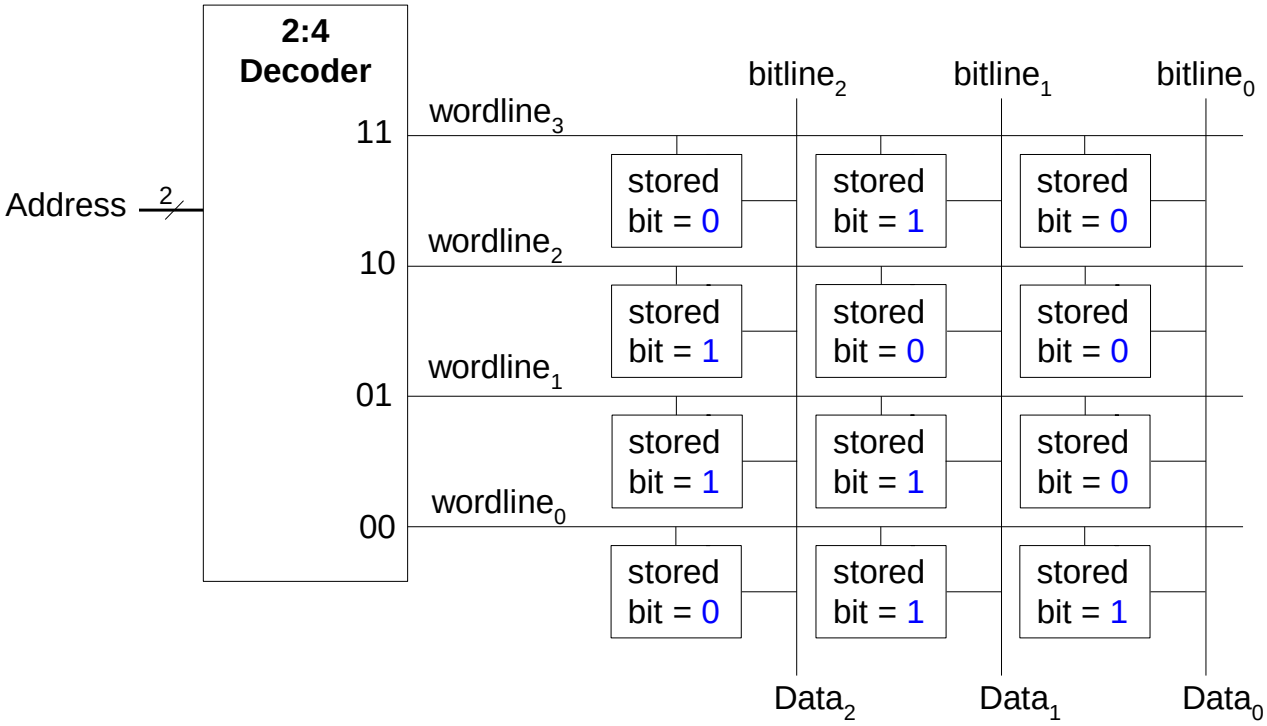

bitline
wordline
stored
bit = 1
+ +
bitline
wordline
stored
bit = 0

SRAM

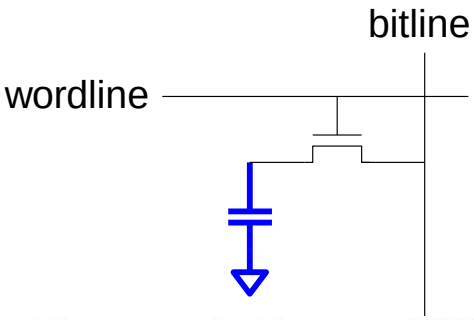




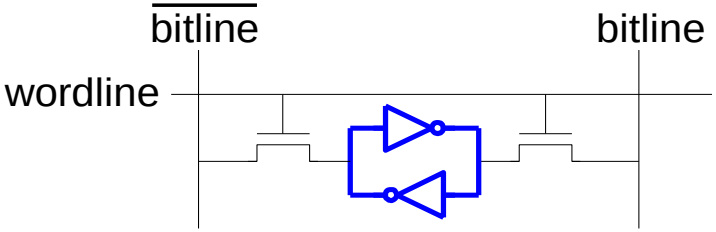
Memory Arrays Review



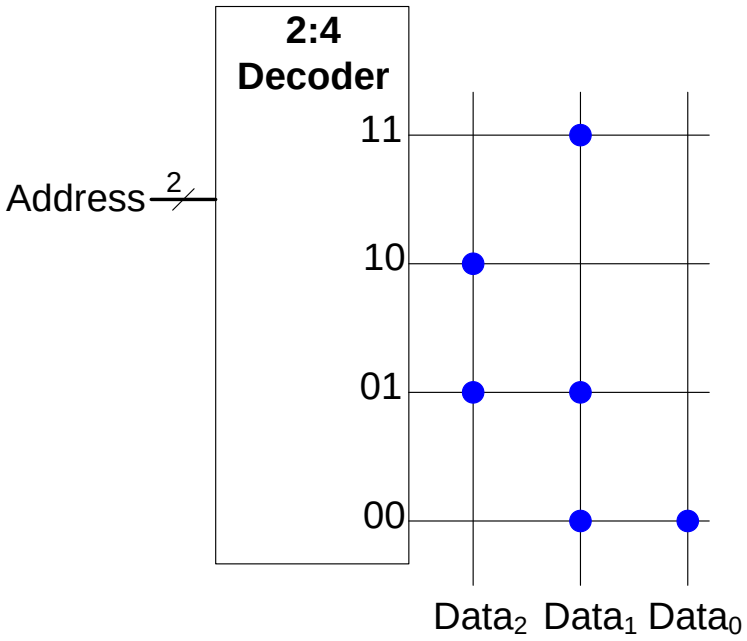
DRAM bit cell:

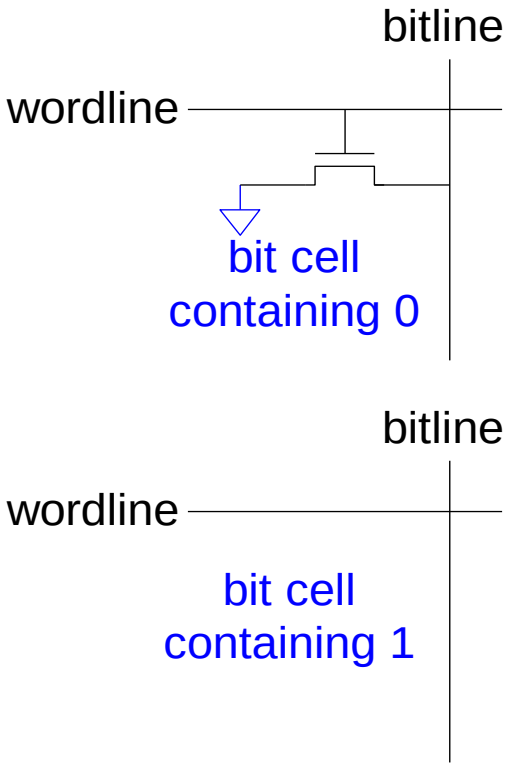


SRAM bit cell:



ROM: Dot Notation

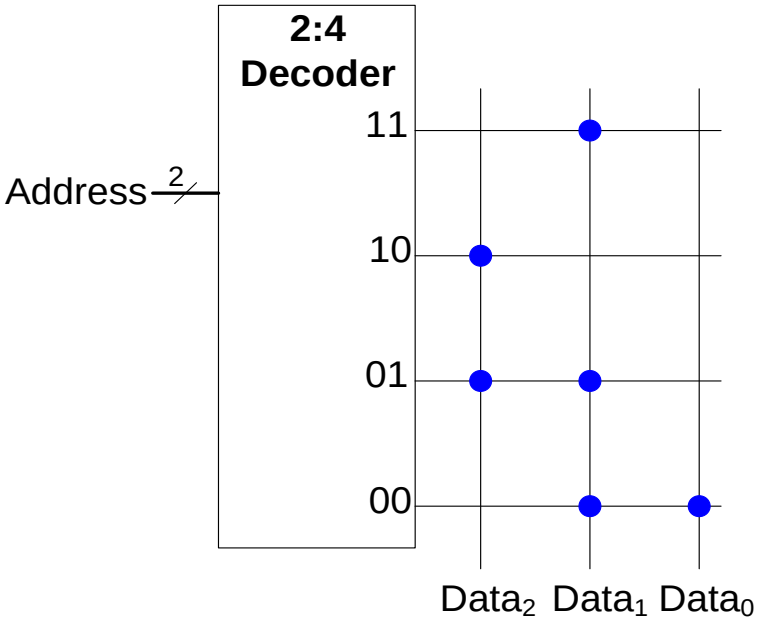




Fujio Masuoka, 1944 -

- Developed memories and high speed circuits at Toshiba, 1971-1994
- Invented Flash memory as an unauthorized project pursued during nights and weekends in the late 1970's
- The process of erasing the memory reminded him of the flash of a camera
- Toshiba slow to commercialize the idea; Intel was first to market in 1988
- Flash has grown into a $25 billion per year market

ROM Storage

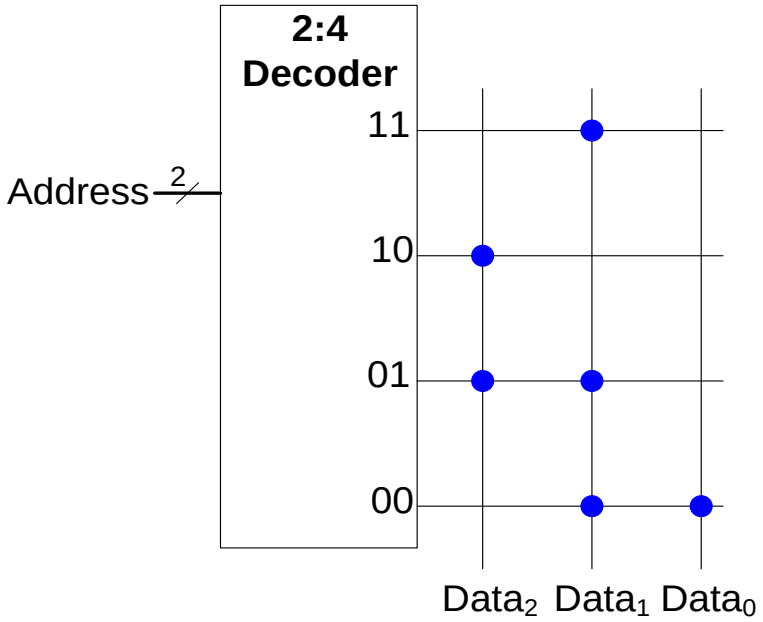


Address	Data		
11	0	1	0
10	1	0	0
01	1	1	0
00	0	1	1

depth

width

ROM Logic



$$Data_2 = A_1 \oplus A_0$$

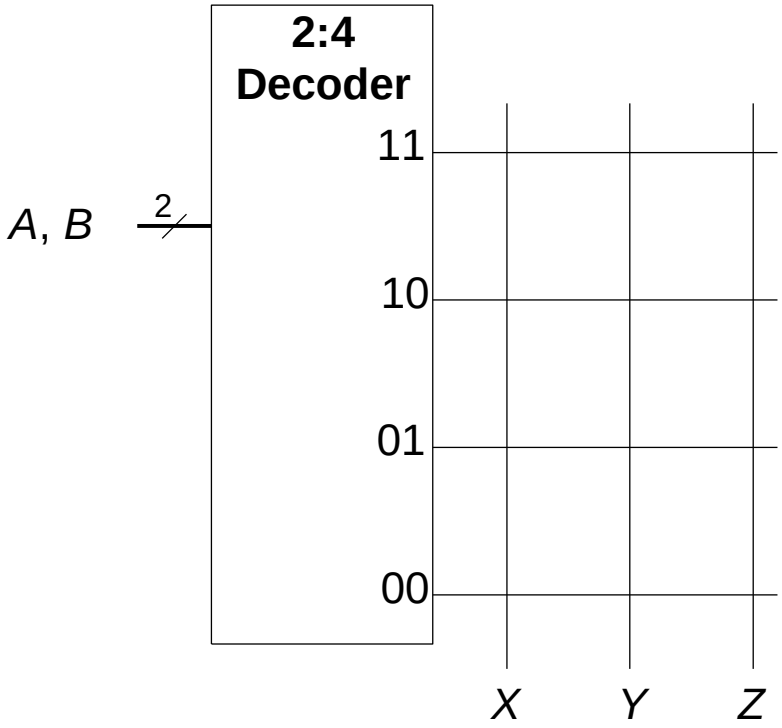
$$Data_1 = \overline{A_1} + A_0$$

$$Data_0 = \overline{A_1}\,\overline{A_0}$$

Example: Logic with ROMs

Implement the following logic functions using a $2^2 \times 3$-bit ROM:

- $X = AB$
- $Y = A + B$
- $Z = A\overline{B}$



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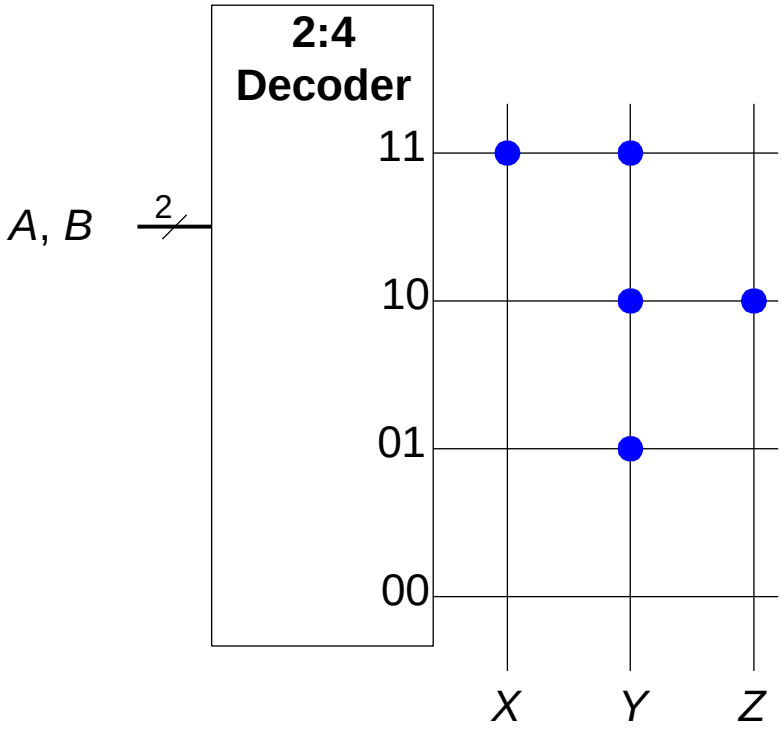
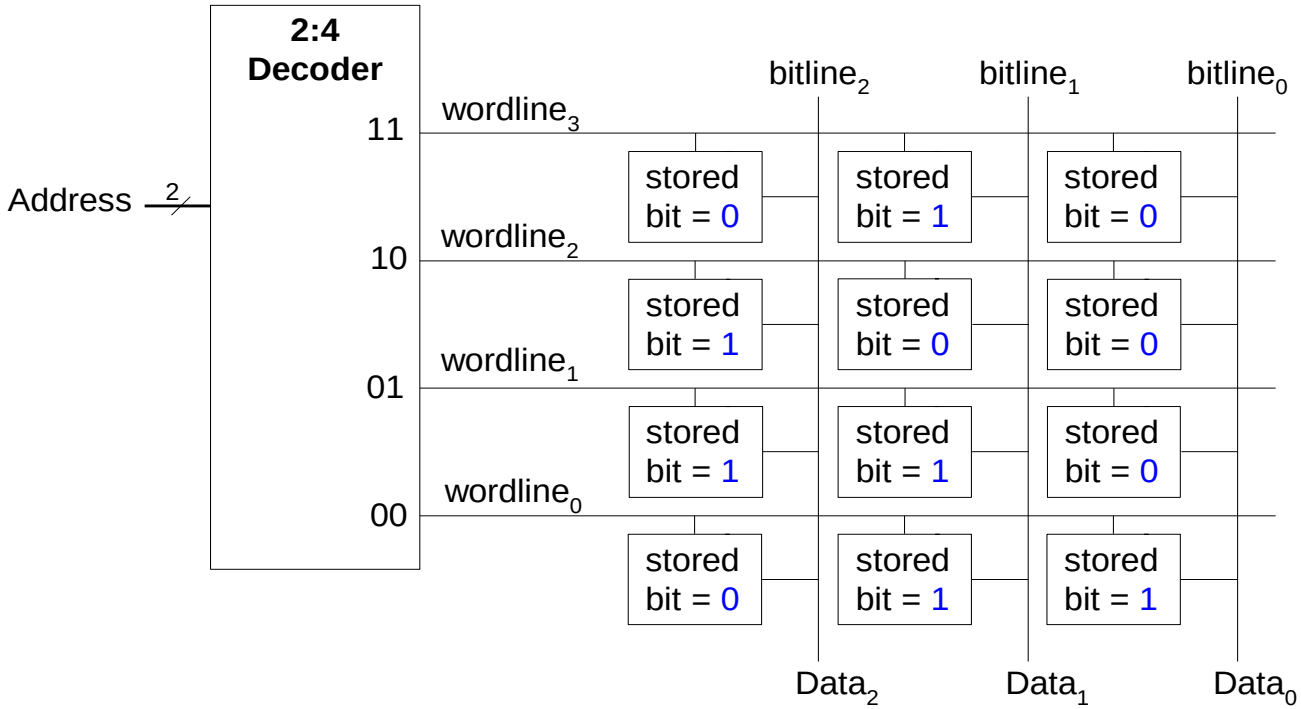


Logic with Any Memory Array



$$Data_2 = A_1 \oplus A_0$$

$$Data_1 = \overline{A}_1 + A_0$$

$$Data_0 = \overline{A}_1 \overline{A}_0$$

Logic with Memory Arrays

Implement the following logic functions using a $2^2 \times 3$-bit memory array:

- $X = AB$
- $Y = A + B$
- $Z = A\overline{B}$

Logic with Memory Arrays

Implement the following logic functions using a $2^2 \times 3$-bit memory array:

- $X = AB$
- $Y = A + B$
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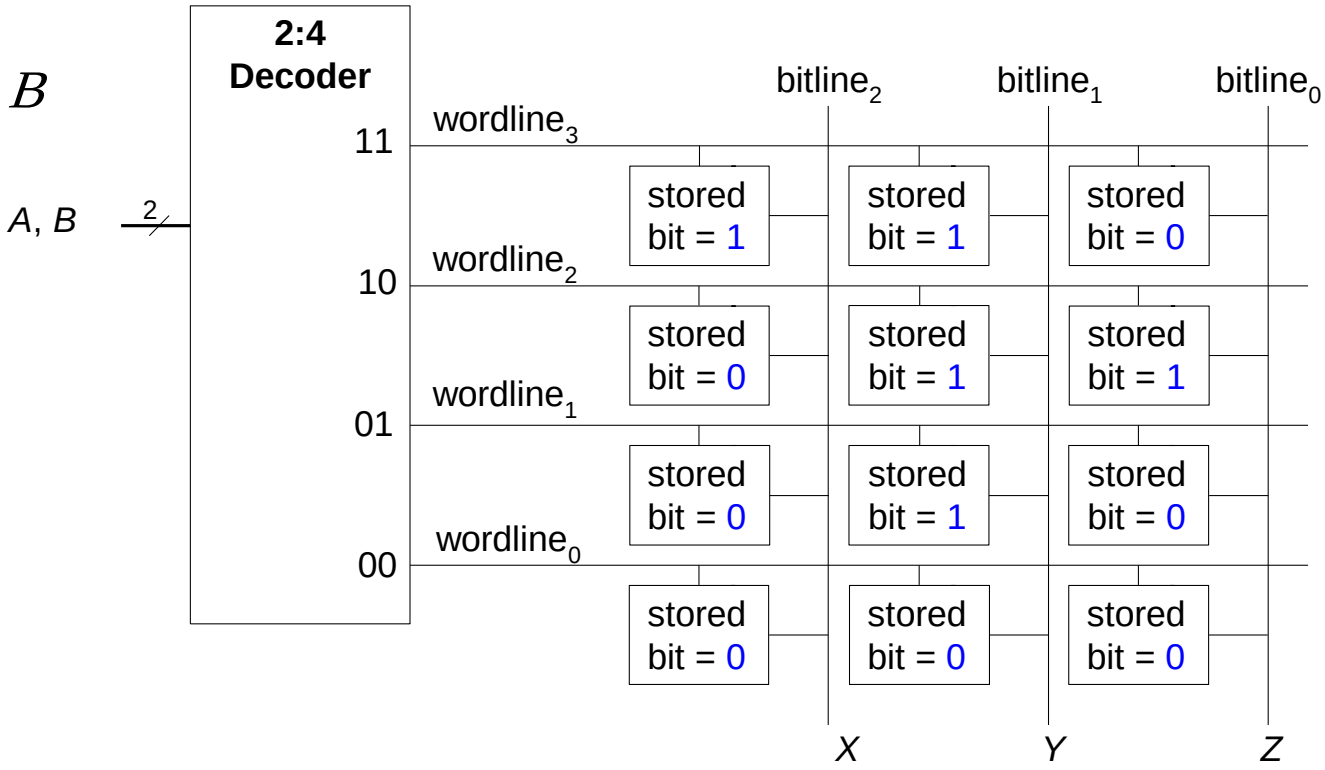
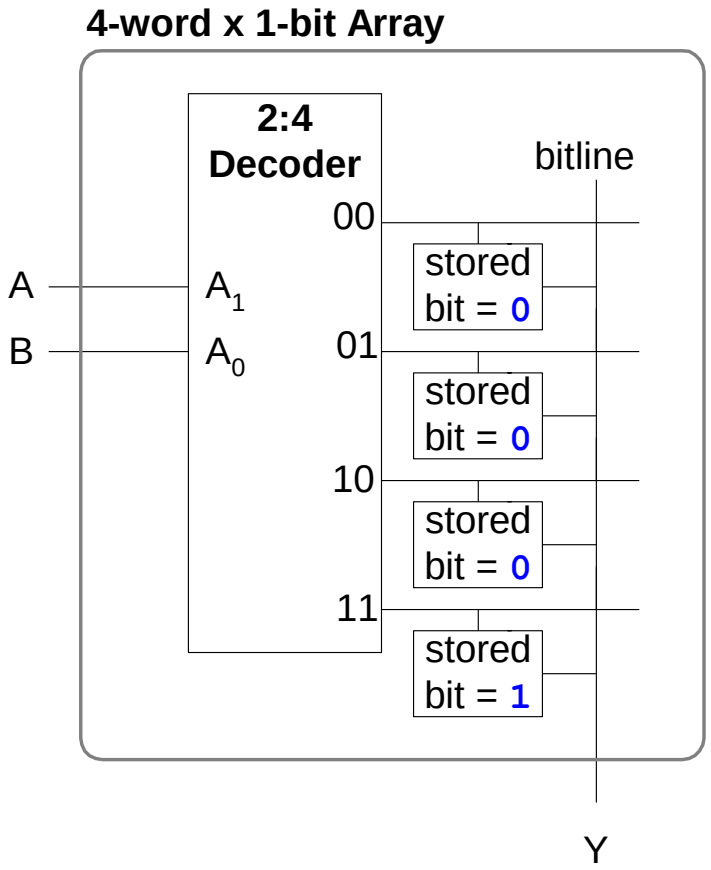


Logic with Memory Arrays

Called *lookup tables* (LUTs): look up output at each input combination (address)

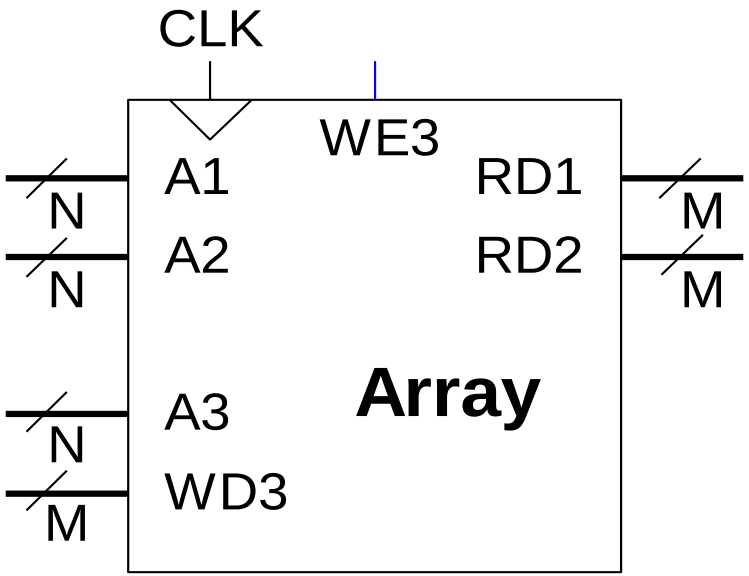
Truth Table

A	B	Y
0	0	**0**
0	1	**0**
1	0	**0**
1	1	**1**



Multi-ported Memories

- **Port:** address/data pair
- 3-ported memory
  - 2 read ports (A1/RD1, A2/RD2)
  - 1 write port (A3/WD3, WE3 enables writing)
- **Register file:** small multi-ported memory



SystemVerilog Memory Arrays

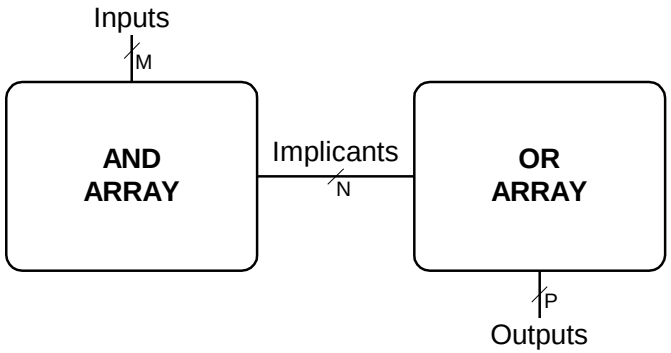
```
// 256 x 3 memory module with one read/write port
module dmem( input  logic       clk, we,
             input  logic [7:0] a,
             input  logic [2:0] wd,
             output logic [2:0] rd);

  logic  [2:0] RAM[255:0];

  assign rd = RAM[a];

  always @(posedge clk)
    if (we)
      RAM[a] <= wd;
endmodule
```

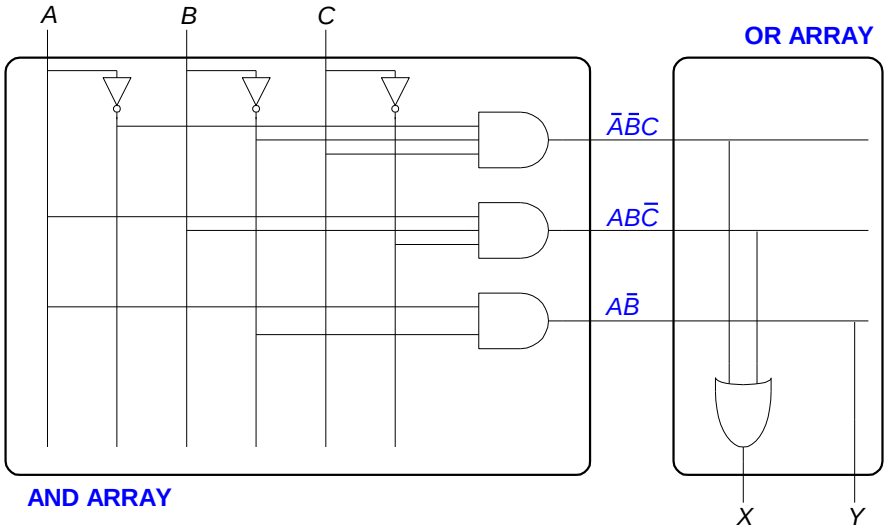
Logic Arrays

- **PLAs** (Programmable logic arrays)
  - AND array followed by OR array
  - Combinational logic only
  - Fixed internal connections
- **FPGAs** (Field programmable gate arrays)
  - Array of Logic Elements (LEs)
  - Combinational and sequential logic
  - Programmable internal connections

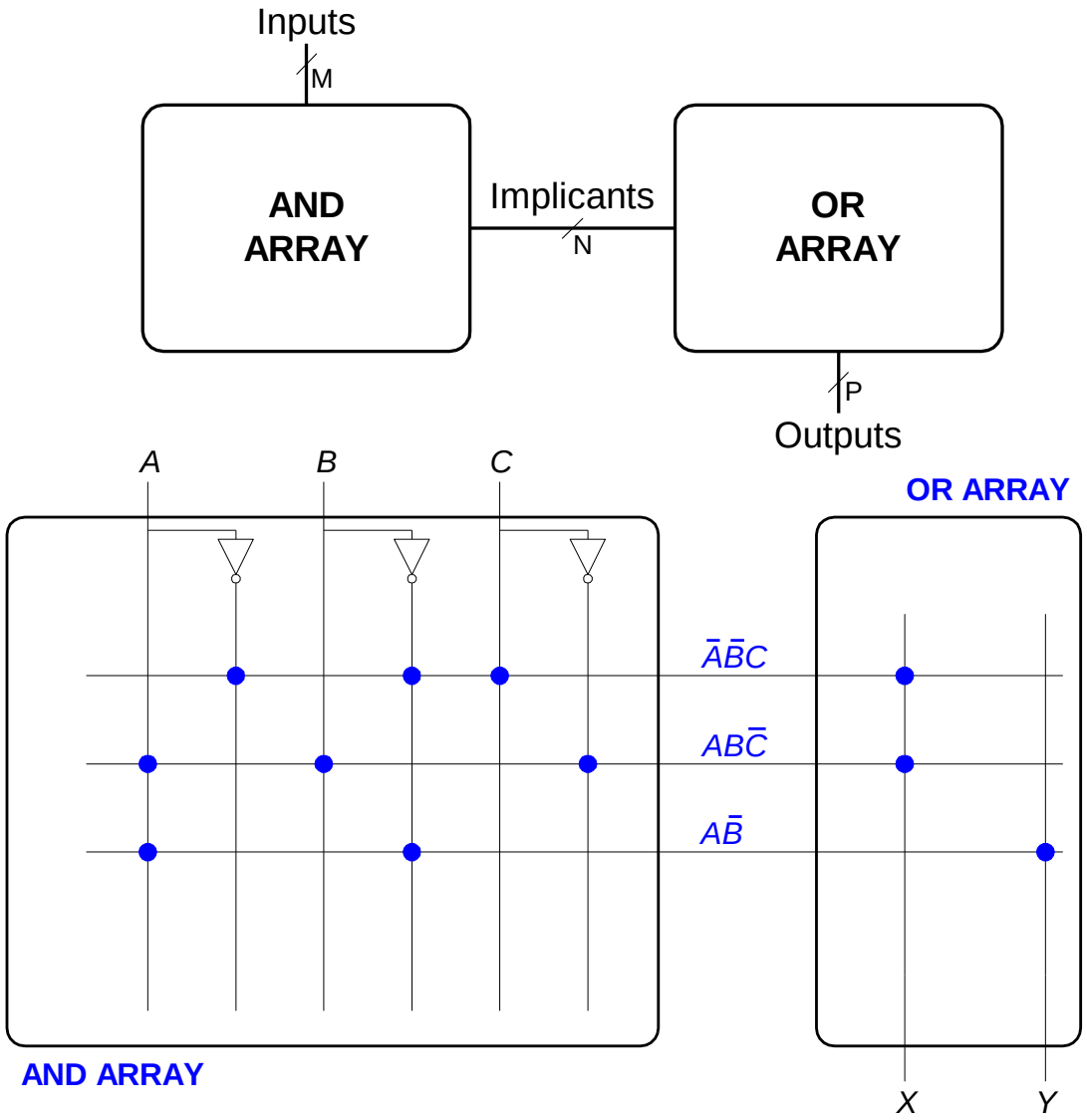
PLAs

- $X = \bar{A}\bar{B}C + AB\bar{C}$
- $Y = A\bar{B}$





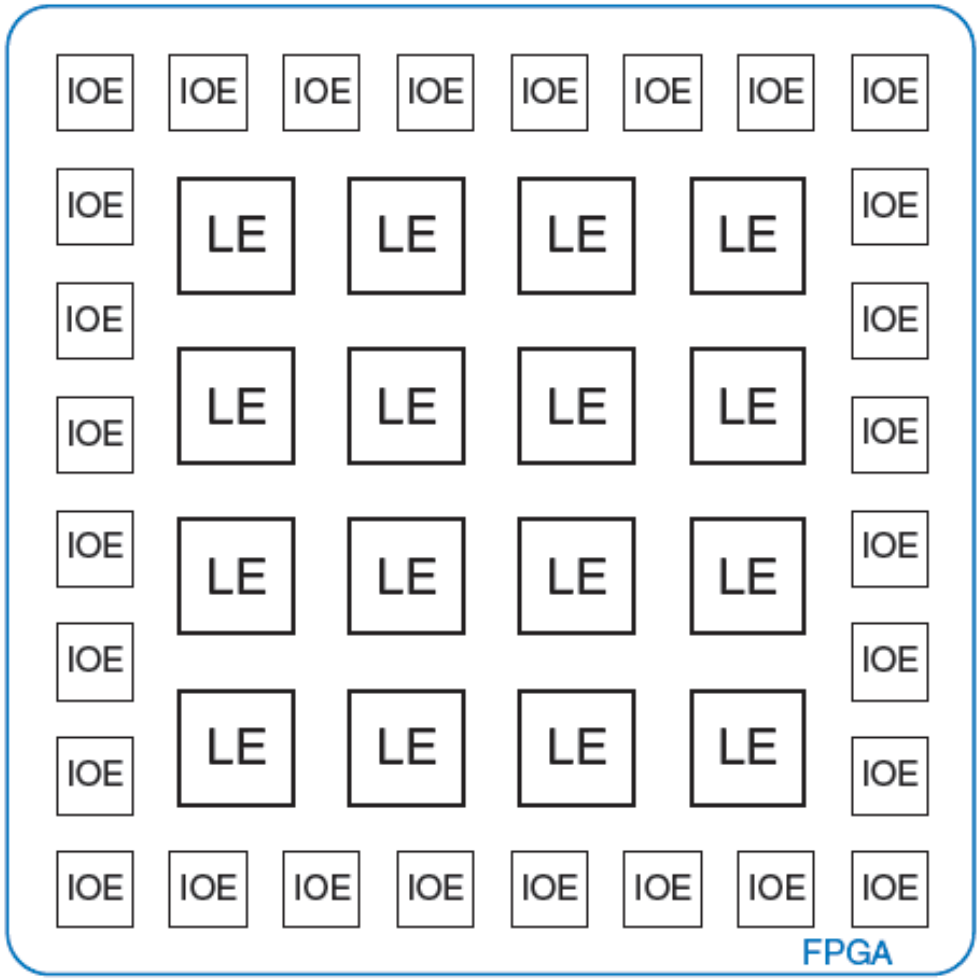
PLAs: Dot Notation



FPGA: Field Programmable Gate Array

- Composed of:
  - **LEs** (Logic elements): perform logic
  - **IOEs** (Input/output elements): interface with outside world
  - **Programmable interconnection:** connect LEs and IOEs
  - Some FPGAs include other building blocks such as multipliers and RAMs

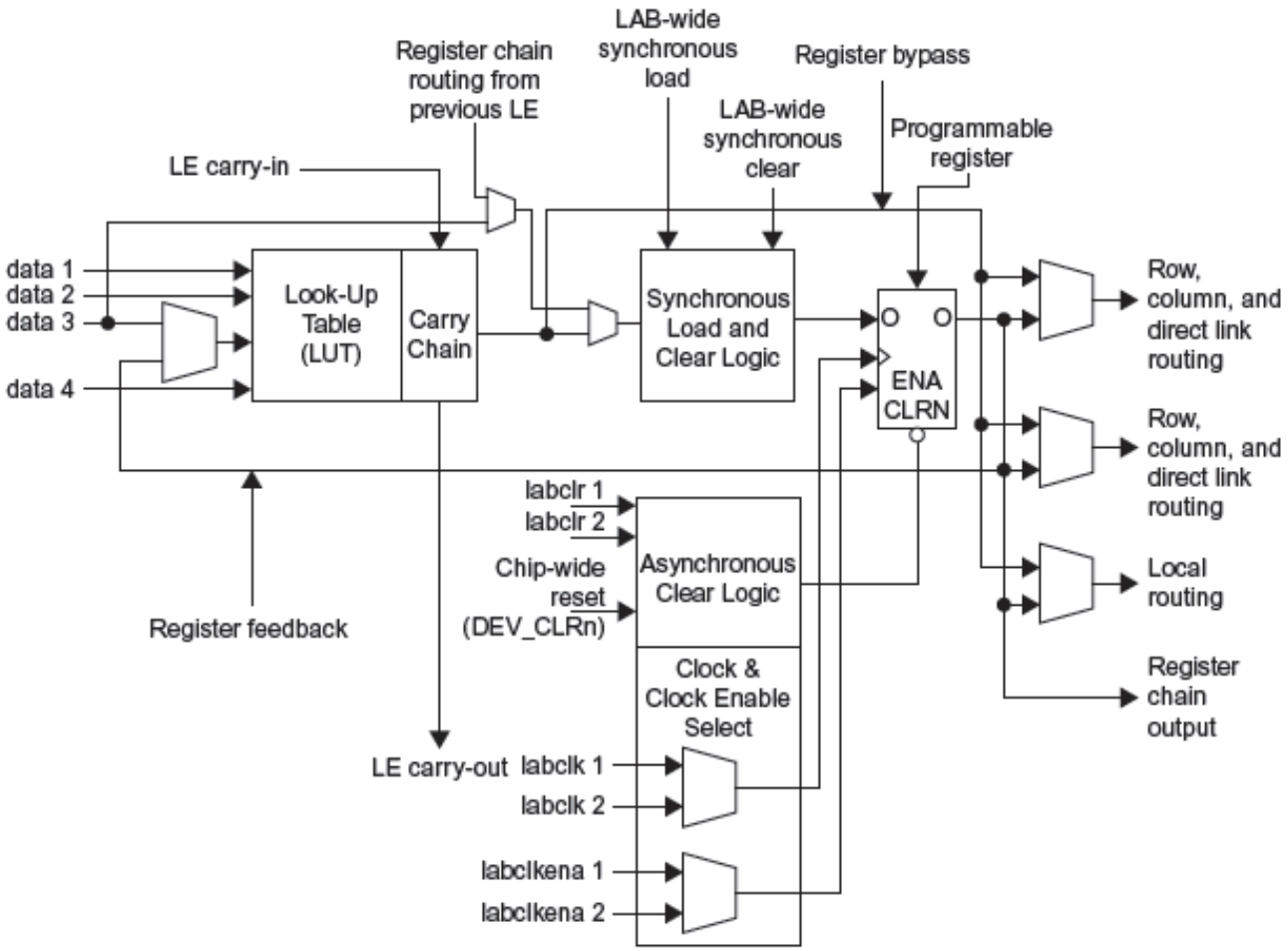
General FPGA Layout



LE: Logic Element

- Composed of:
  - **LUTs** (lookup tables): perform combinational logic
  - **Flip-flops:** perform sequential logic
  - **Multiplexers:** connect LUTs and flip-flops

Altera Cyclone IV LE



Altera Cyclone IV LE

- The Altera Cyclone IV LE has:
  - 1 four-input LUT
  - 1 registered output
  - 1 combinational output

LE Configuration Example

Show how to configure a Cyclone IV LE to perform the following functions:

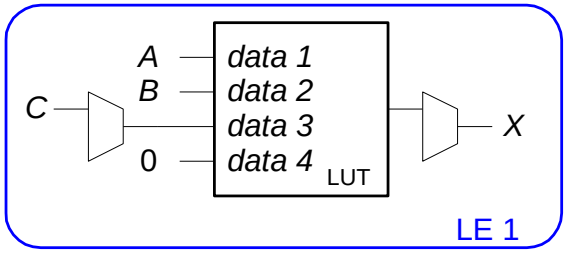
- $X = \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C}$
- $Y = A\overline{B}$

LE Configuration Example

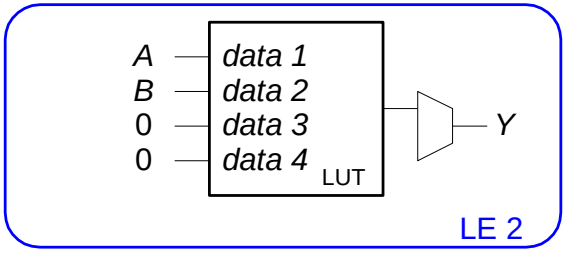
Show how to configure a Cyclone IV LE to perform the following functions:

- $X = \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C}$
- $Y = A\overline{B}$

(A) data 1	(B) data 2	(C) data 3	data 4	(X) LUT output
0	0	0	X	0
0	0	1	X	1
0	1	0	X	0
0	1	1	X	0
1	0	0	X	0
1	0	1	X	0
1	1	0	X	1
1	1	1	X	0



(A) data 1	(B) data 2	data 3	data 4	(Y) LUT output
0	0	X	X	0
0	1	X	X	0
1	0	X	X	1
1	1	X	X	0



FPGA Design Flow

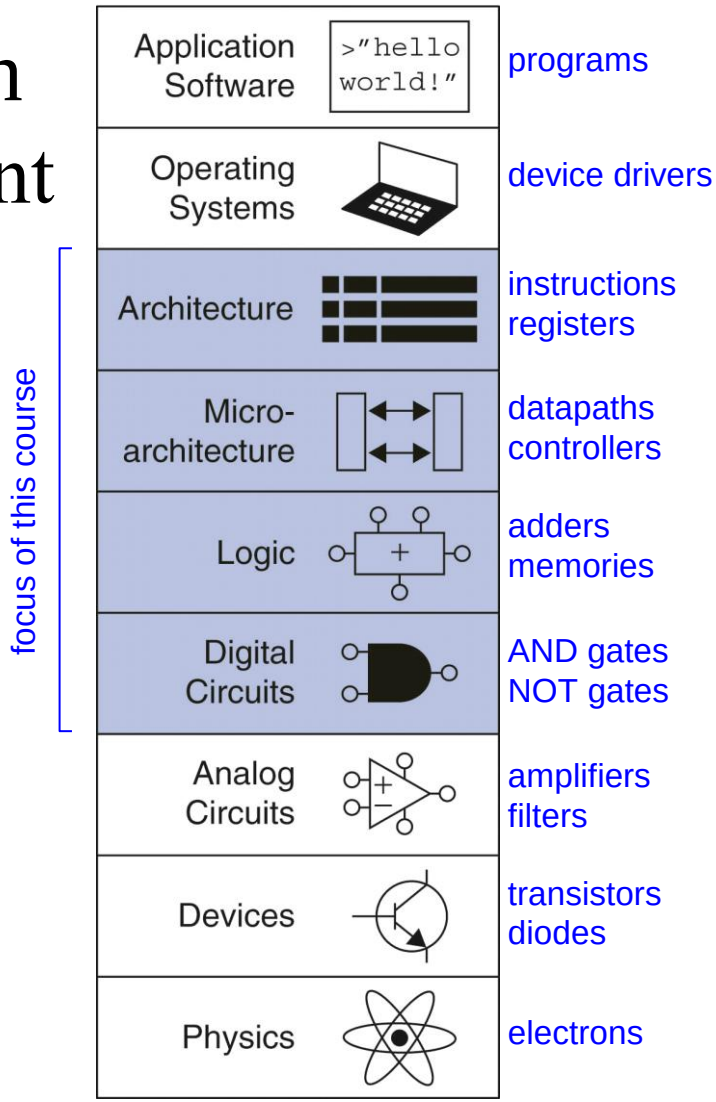
Using a CAD tool (such as Altera's Quartus II)

- **Enter the design** using schematic entry or an HDL
- **Simulate** the design
- **Synthesize** design and map it onto FPGA
- **Download the configuration** onto the FPGA
- **Test** the design

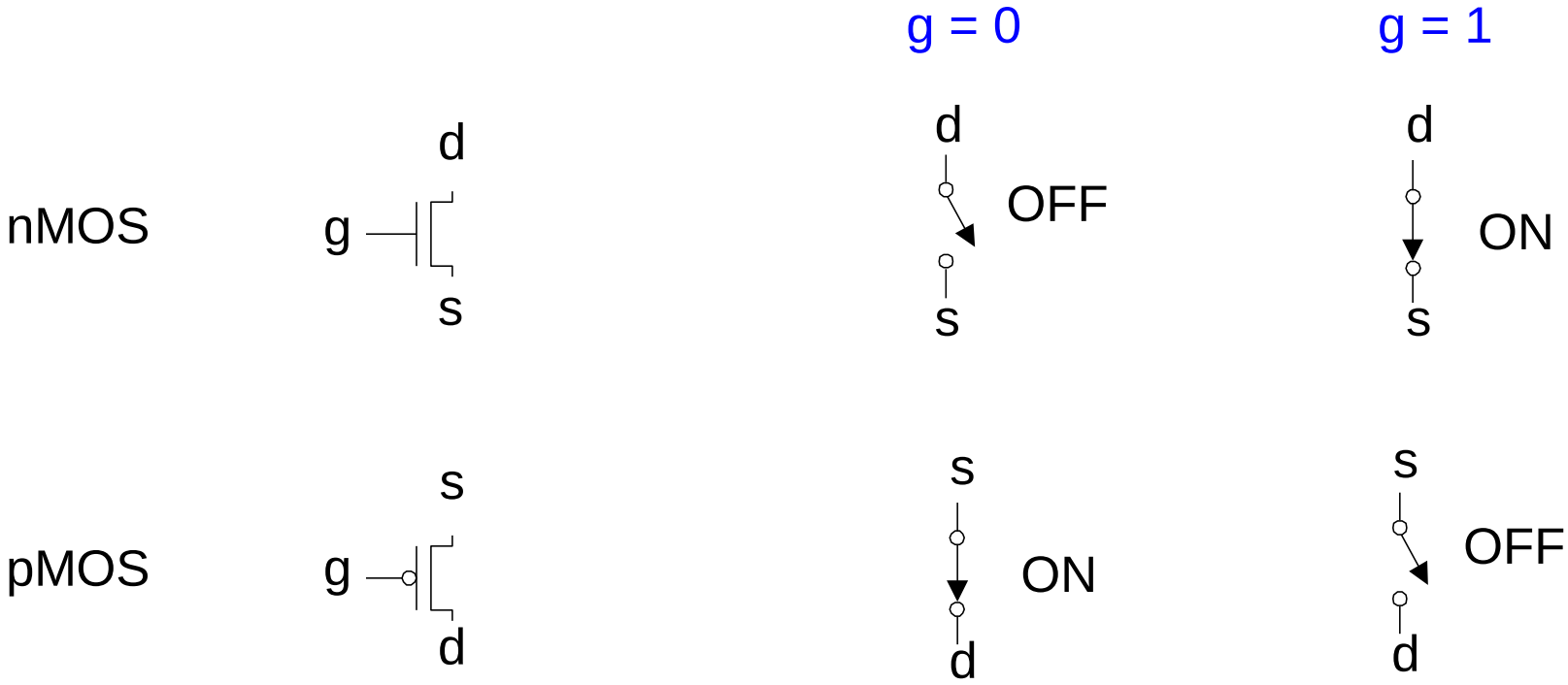
Quick Review
Previous Chapters

Abstraction

- Hiding details when they aren't important



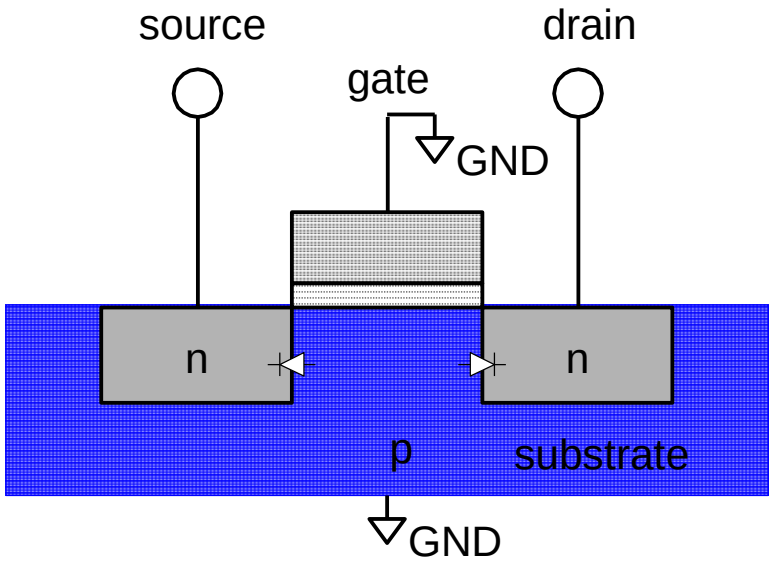
Transistor Function



Transistors: nMOS

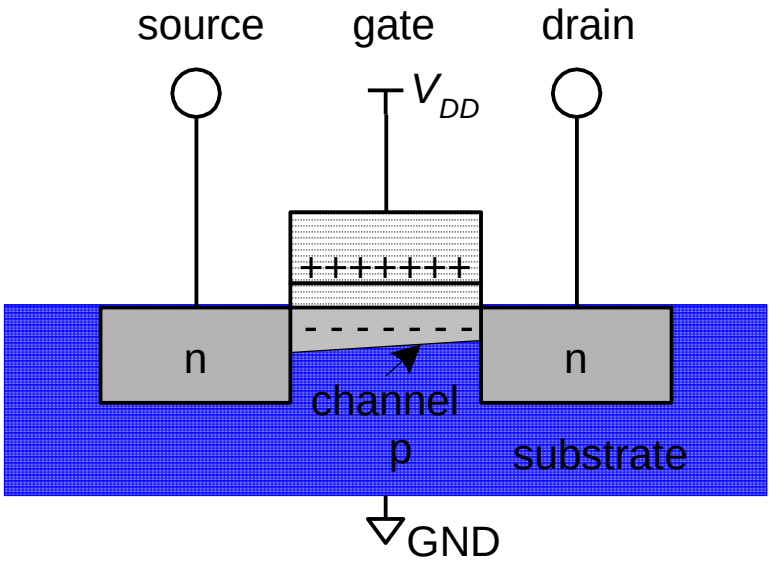
Gate = 0

OFF (no connection between source and drain)



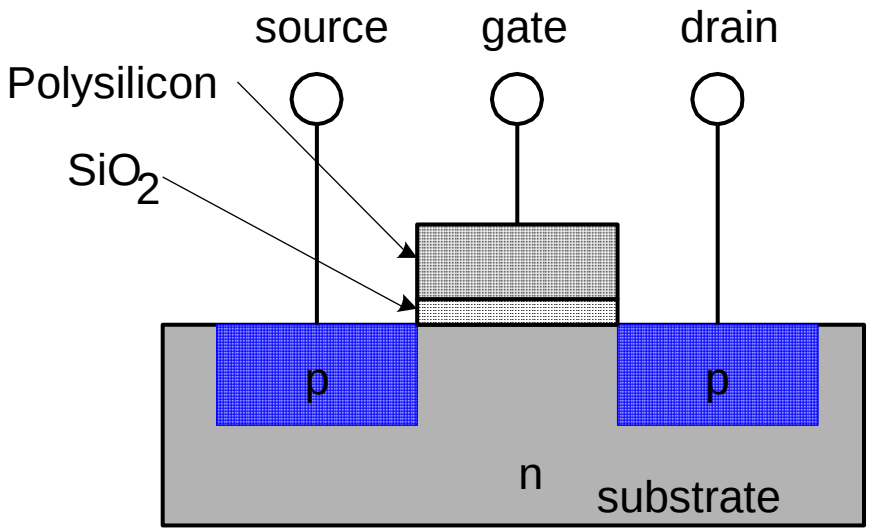
Gate = 1

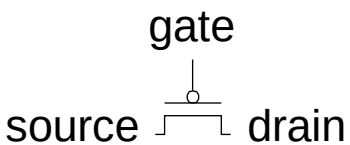
ON (channel between source and drain)



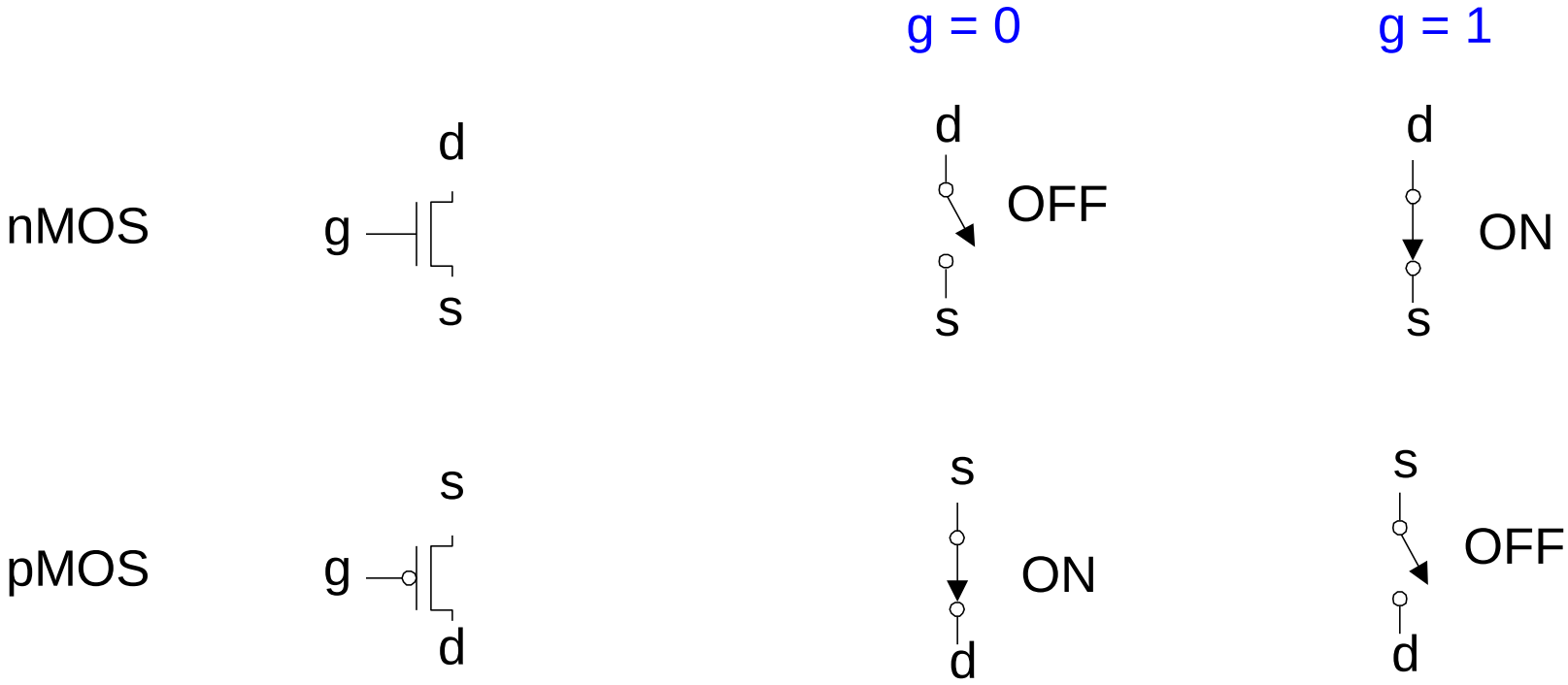
Transistors: pMOS

- pMOS transistor is opposite
  - ON when Gate = 0
  - OFF when Gate = 1



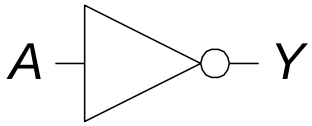


Transistor Function



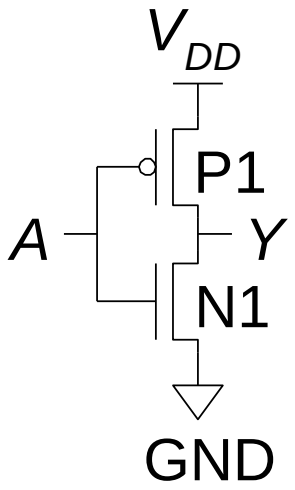
CMOS Gates: NOT Gate

NOT



$Y = \overline{A}$

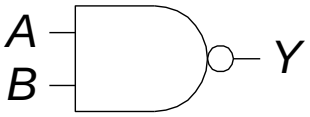
A	Y
0	1
1	0



A	P1	N1	*Y*
0	ON	OFF	1
1	OFF	ON	0

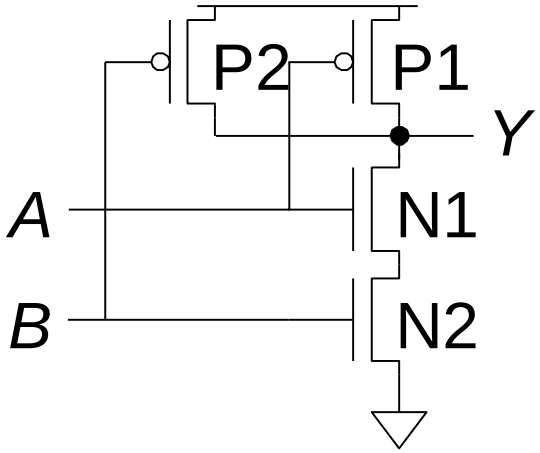
CMOS Gates: NAND Gate

NAND



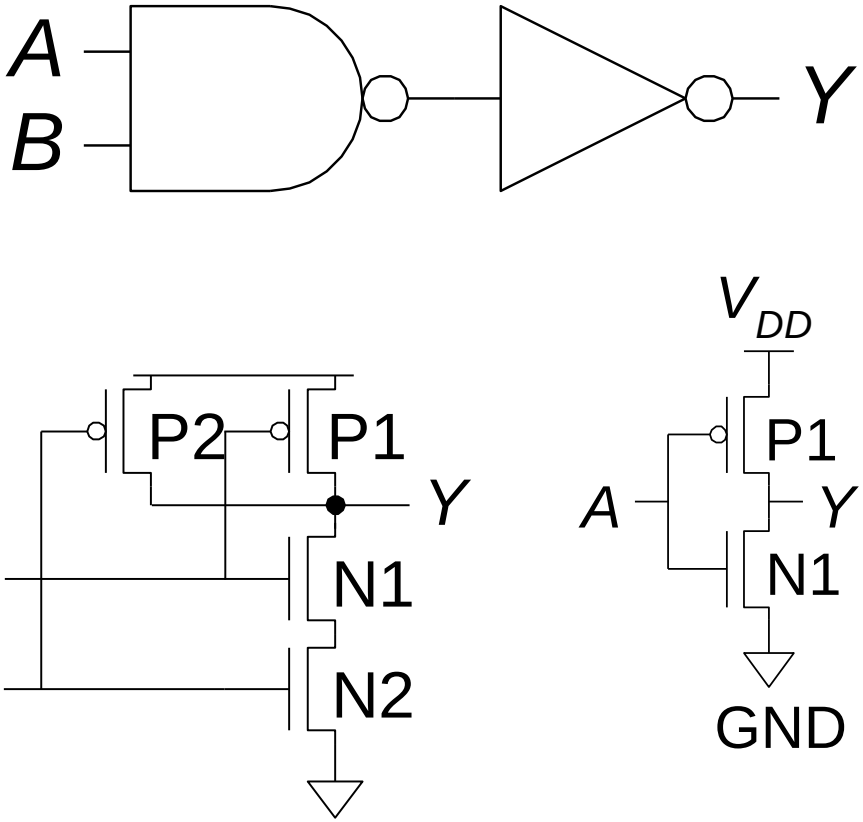
$Y = \overline{AB}$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



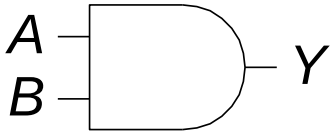
A	B	P1	P2	N1	N2	Y
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	1
1	0	OFF	ON	ON	OFF	1
1	1	OFF	OFF	ON	ON	0

AND Gate



Two-Input Logic Gates

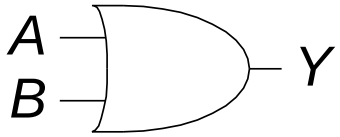
AND



$Y = AB$

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR

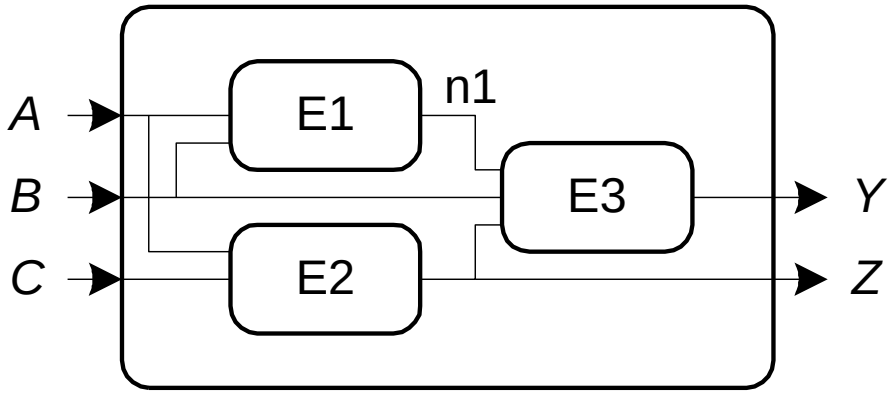


$Y = A + B$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

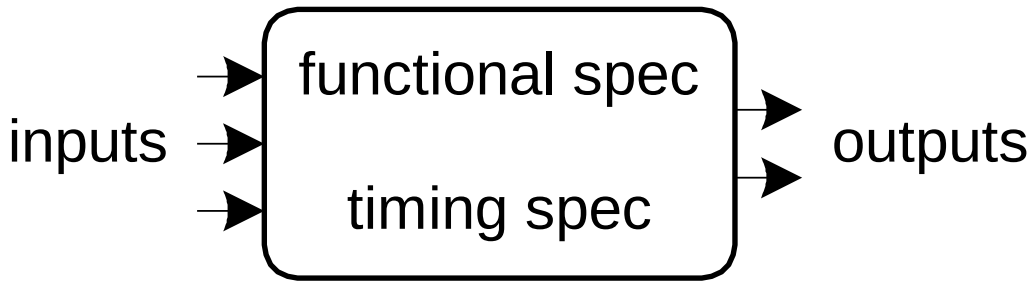
Circuits

- Nodes
  - Inputs: *A*, *B*, *C*
  - Outputs: *Y*, *Z*
  - Internal: n1
- Circuit elements
  - E1, E2, E3
  - Each a circuit



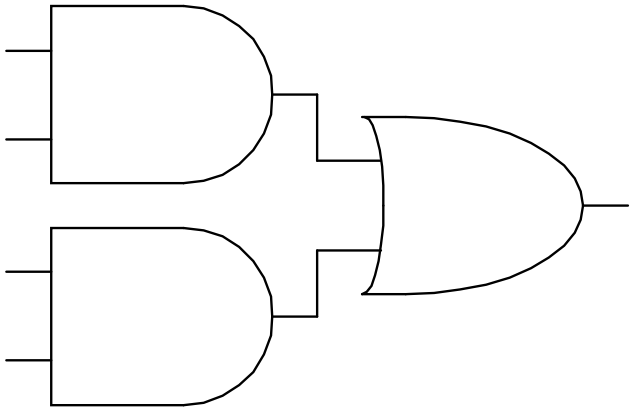
Types of Logic Circuits

- **Combinational Logic**
  - Memoryless
  - Outputs determined by current values of inputs
- **Sequential Logic**
  - Has memory
  - Outputs determined by previous and current values of inputs

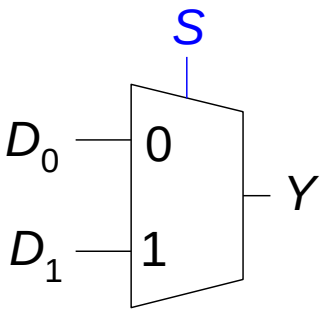


Combinational Composition

- Every element is combinational
- Every node is either an input or connects to *exactly one* output
- The circuit contains no cyclic paths
- **Example:**

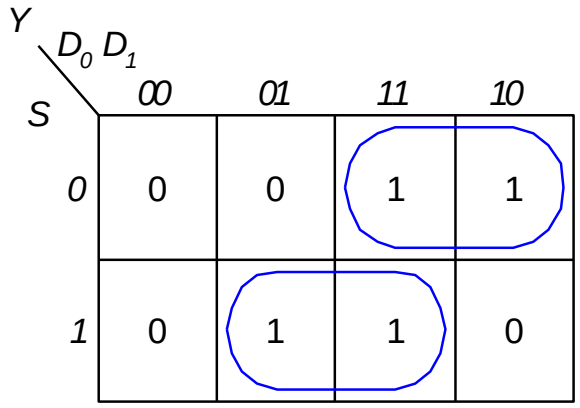


Multiplexer

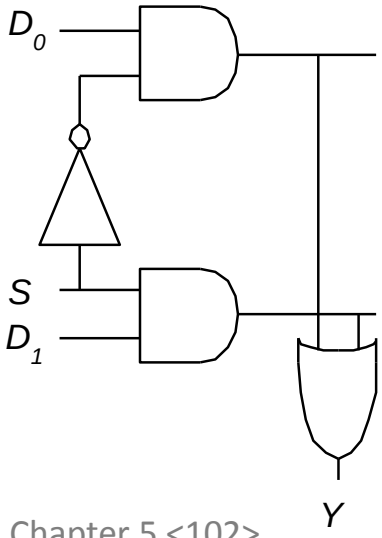


S	D_1	D_0	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

S	Y
0	D_0
1	D_1



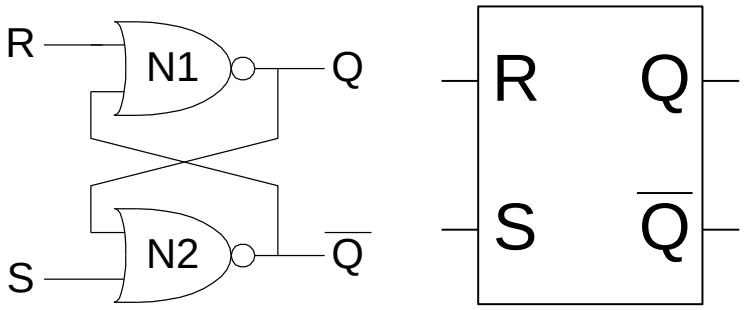
$$Y = D_0\overline{S} + D_1S$$



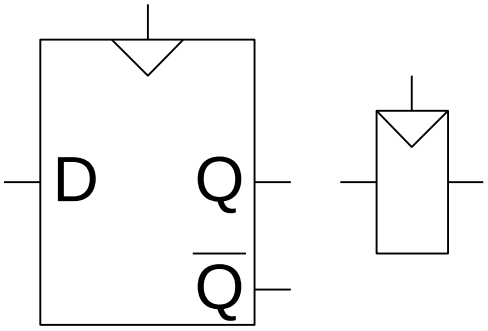
Sequential Circuits

- Give sequence to events
- Have memory (short-term)
- Use feedback from output to input to store information

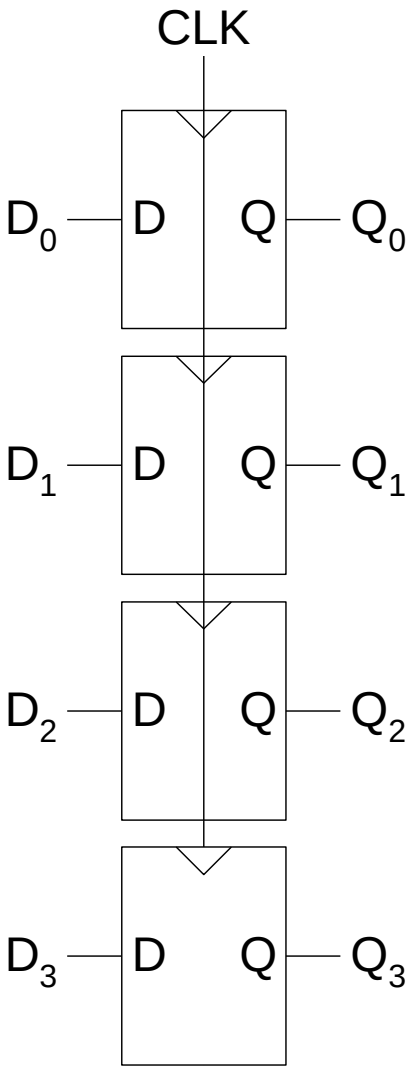
SR Latch Symbol

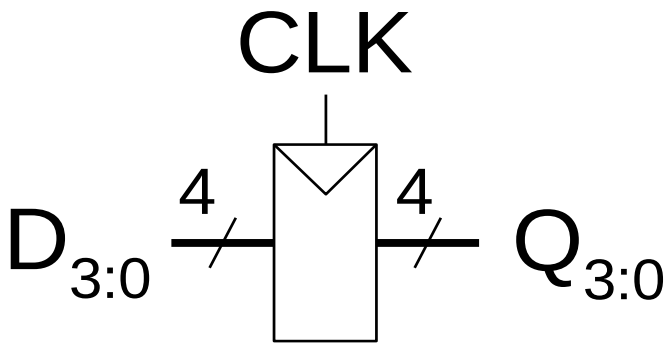


D Flip-Flop Symbols



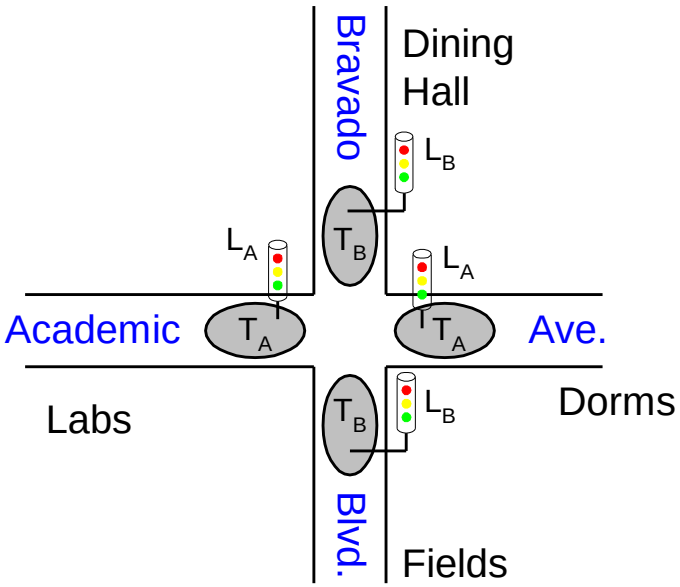
Registers

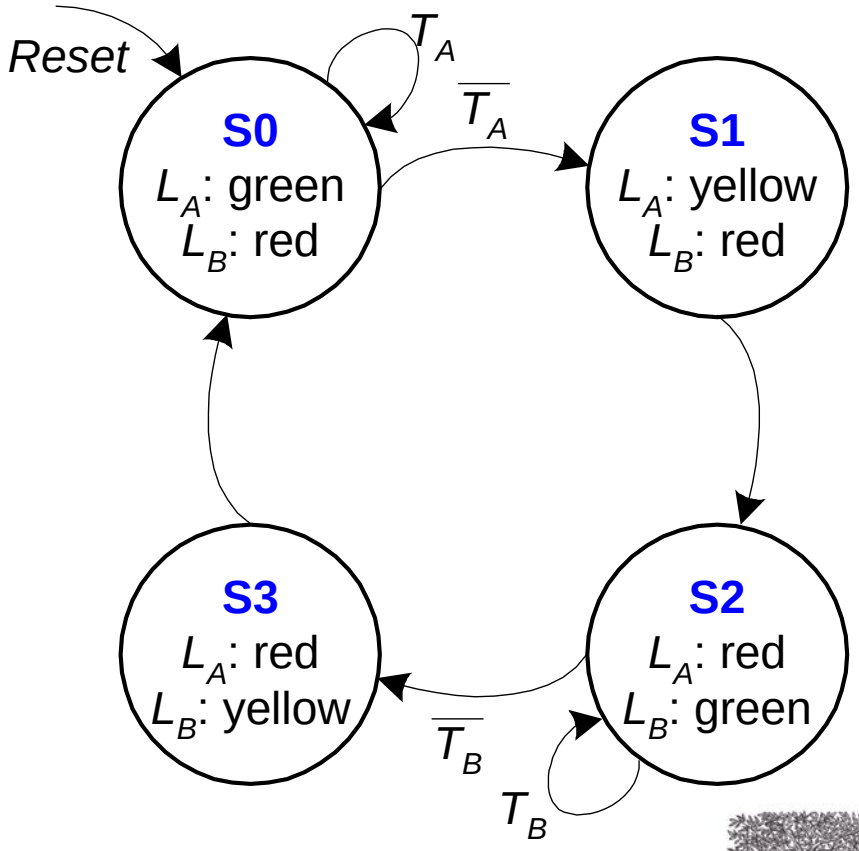




FSM State Transition

- **Moore FSM:** outputs labeled in each state
- **States:** Circles
- **Transitions:** Arcs

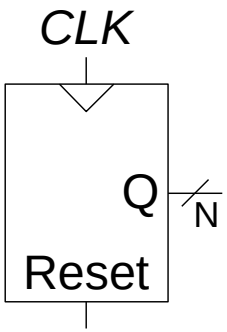


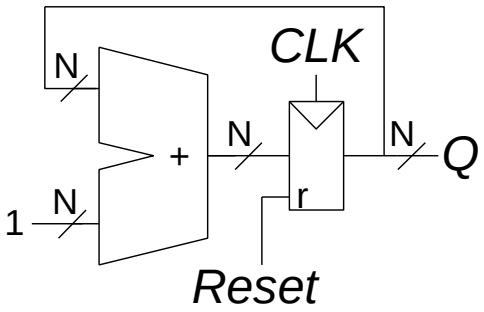


Counters

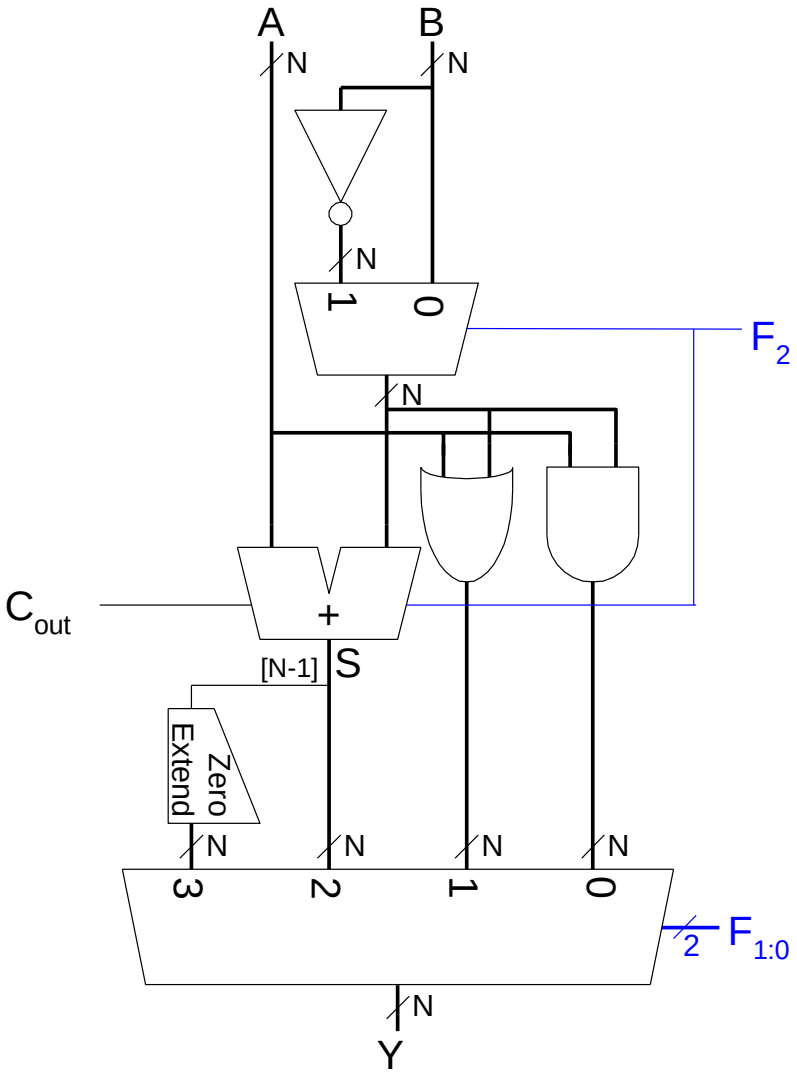
- Increments on each clock edge
- Used to cycle through numbers. For example,
  - 000, 001, 010, 011, 100, 101, 110, 111, 000, 001…
- Example uses:
  - Digital clock displays
  - Program counter: keeps track of current instruction executing

Symbol **Implementation**





ALU Design



$F_{2:0}$	Function
000	A & B
001	A | B
010	A + B
011	not used
100	A & ~B
101	A | ~B
110	A - B
111	SLT

Divider

$$
\begin{array}{rrrr|l}
 & 1 & 5 & 1 & 2 \\
\hline
3) & 4 & 5 & 3 & 7 \\
 & 3 & & & \\
\hline
 & 1 & 5 & & \\
 & 1 & 5 & & \\
\hline
 & & & 3 & \\
 & & & 3 & \\
\hline
 & & & & 7 \\
 & & & & 6 \\
\hline
 & & & & 1
\end{array}
\qquad
\begin{array}{l}
1512 \leftarrow \text{Quotient} \\
\overline{3)4537} \leftarrow \text{Dividend} \\
3000 \leftarrow \text{Divisor} * q(\text{MSD}) * 10^3 \\
\overline{1537} \leftarrow \text{Partial remainder} \\
1500 \\
\overline{0037} \\
0030 \\
\overline{0007} \\
0006 \\
\overline{0001} \leftarrow \text{Remainder}
\end{array}
$$

$$
\begin{array}{llll}
4537 - 1 * 3 * 10^3 = 1537 & \text{or} & R(4) - q_3 * D * 10^3 = R(3) \\
1537 - 5 * 3 * 10^2 = 0037 & \text{or} & R(3) - q_2 * D * 10^2 = R(2) \\
0037 - 1 * 3 * 10^1 = 0007 & \text{or} & R(2) - q_1 * D * 10^1 = R(1)' \\
0007 - 2 * 3 * 10^0 = 0001 & \text{or} & R(1) - q_0 * D * 10^0 = R(0)
\end{array}
$$

or, in general, at any step:

$$\boxed{R(i) = R(i+1) - q_i * D * 10^i},$$

where $i = n-1, n-2, \ldots, 1, 0.$

Divider

$$
\begin{array}{rlll}
 & 4537 - 3 * 10^3 = +1537 & & q_3 = 1 \\
 & 1537 - 3 * 10^3 = -1463 & & q_3 = 2 \\
 & -1463 + 3 * 10^3 = +1537 & \text{restore} & \boxed{q_3 = 1} \\
 & +1537 - 3 * 10^2 = +1237 & & q_2 = 1 \\
 & +1237 - 3 * 10^2 = +937 & & q_2 = 2 \\
 & +\ 937 - 3 * 10^2 = +637 & & q_2 = 3 \\
 & +\ 637 - 3 * 10^2 = +337 & & q_2 = 4 \\
 & +\ 337 - 3 * 10^2 = +37 & & q_2 = 5 \\
 & +\ 37 - 3 * 10^2 = -263 & & q_2 = 6 \\
 & -\ 263 + 3 * 10^2 = +37 & \text{restore} & \boxed{q_2 = 5} \\
 & +\ 37 - 3 * 10^1 = +7 & & q_1 = 1 \\
 & +\ 7 - 3 * 10^1 = -23 & & q_1 = 2 \\
 & -\ 23 + 3 * 10^1 = +7 & \text{restore} & \boxed{q_1 = 1} \\
 & +\ 7 - 3 * 10^0 = +4 & & q_0 = 1 \\
 & +\ 4 - 3 * 10^0 = +1 & & q_0 = 2 \\
 & +\ 1 - 3 * 10^0 = -2 & & q_0 = 3 \\
 & -\ 2 + 3 * 10^0 = +1 & \text{restore} & \boxed{q_0 = 2}
\end{array}
$$

Restoring for a binary division 29/3

$$
\begin{array}{rlll}
29 - 3 * 2^4 = -19 & & & q_4 = 1 \\
-19 + 3 * 2^4 = +29 & \text{restore} & & \boxed{q_4 = 0} \\
29 - 3 * 2^3 = +5 & & & \boxed{q_3 = 1} \\
+5 - 3 * 2^2 = -7 & & & q_2 = 1 \\
-7 + 3 * 2^2 = +5 & \text{restore} & & \boxed{q_2 = 0} \\
+5 - 3 * 2^1 = -1 & & & q_1 = 1 \\
-1 + 3 * 2^1 = +5 & \text{restore} & & \boxed{q_1 = 0} \\
+5 - 3 * 2^0 = +2 & & & \boxed{q_0 = 1}
\end{array}
$$